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Innovative ICT Solutions
for the Societal Challenges

New Approaches in Reliability Analysis of Complex Systems

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Some Intro

Slovakia and Žilina Region

Žilina is a regional city and has near 85 thousands inhabitants.

University of Žilina

- Technical university
- Established in 1953
- About 9000 students and 1500 employees
- More than 70,000 graduates
- Main area of research – transportation
- 7 faculties:
  - **Faculty of Management Science and Informatics**

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Faculty of Management Science and Informatics

- Study programs:
  - informatics, computer engineering, management
- Established in 1990
- About 1500 students and 140 employees
- More than 3500 graduates
- Main area of research – optimization of (transport) networks, decision support systems, biomedicine
- 7 departments:
  - **Department of Informatics** - around 15 academics and research fellows who form research community in Computer Science.

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Our Team

Data Mining

- Decision Making Support Systems
- Fuzzy Decision Trees
- Clustering and Classification

Reliability Engineering

- Reliability Analysis
- Importance Measures
- Sensitivity and Testability

Application in

- medicine
- decision systems

our Projects:

- FP7-ICT-2013-10. Regional Anesthesia Simulator & Assistant (RASimAs), Reg. no.610425, 2013-2016
- Support Systems for Medical Decision Making, Grant of Research & Development Agency (APVV), Reg. no. SK-PL-0023-12, Slovakia-Poland, 2013-2014
- Workshop on Biomedical Technologies, Grant of Visehrad Fund V4, 2014
- Intelligent Assistance Systems: Multisensor Processing and Reliability Analysis, NATO Collaborative Linkage Grant, Reg.. no. CBP.EAP.CLG 984, 2011-2012
- TEMPUS. Advanced Training and Life Long Learning Program in Applied Health Sciences, Reg. No. 543889-TEMPUS-1-2013-1-SE, 2013-2016
- TEMPUS. Green Computing and Communications (GreenCo), Reg.No.530270-TEMPUS-1-2012-1-UK, 2012-2015

etc.

Our Cooperation

- Institute of Biomedical Informatics, University of Information Technology and Management in Rzeszów, *Poland* (Dr. Krzysztof Pancerz)
- VŠB - Technical university of Ostrava, *Czech Republic* (Prof. Radim Briš, CSc.)
- University Medical Centre Utrecht Image Sciences Institute, *The Netherlands* (Prof. Max A. Viergever)
- Aachen University of Technology, Department of Medical Informatics, *Germany* (Prof. Thomas M. Deserno)

- United Institute of Informatics Problems, *Belarus* (Prof. Alexander Tuzikov)
- Siberian State Medical University, *Russia* (Prof. Sergey Karas)
- Bay Zoltán Nonprofit Ltd., *Hungary*, (Dr. Balint Uzsoki)
- University of Ioannina*, Greece* (Dr. Iosif Androulidakis)
- Università Campus Bio-Medico di Roma*, Italy* (Prof. Paolo Soda)

Safety of Healthcare

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Reliability Analysis

System

Mathematical model

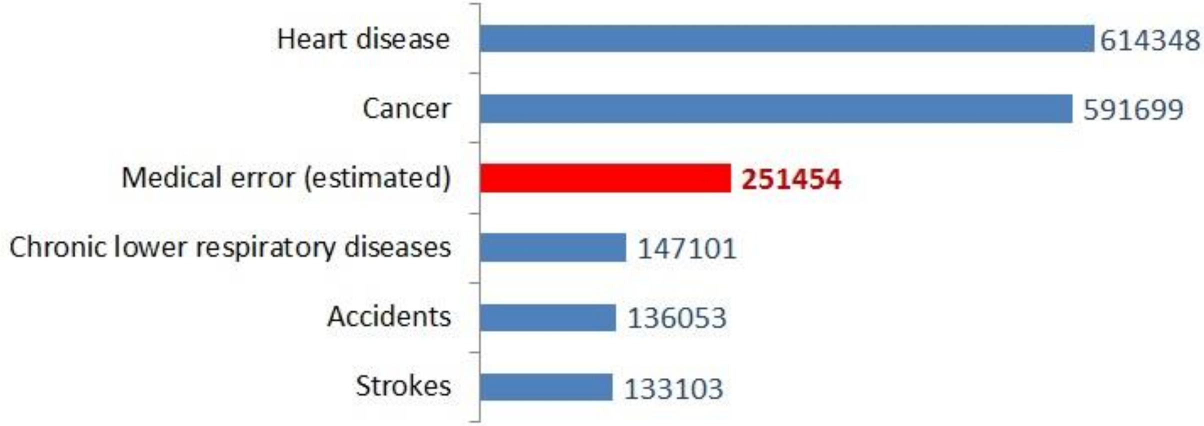
Reliability indices and measures

- *System availability*
- *MTTF, MTBF…*
- *Reliability function*
- *Importance measures*
- …

System improving

Safety of Healthcare

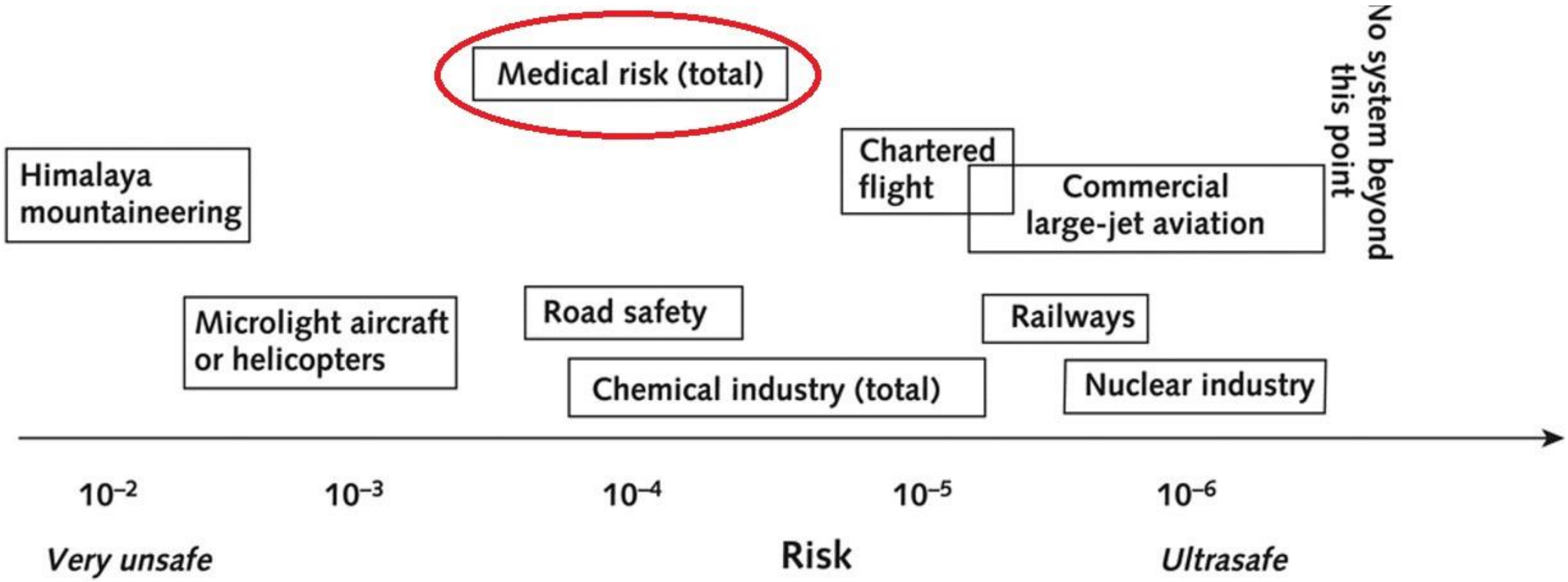
- Medical error is one of the leading causes of death in the US.



- About **8 – 12% of patients** admitted to hospital suffer from **adverse events** whilst receiving healthcare in the EU.

- M. A. Makary and M. Daniel, "Medical error—the third leading cause of death in the US," BMJ, vol. 353, p. i2139, May 2016
- https://ec.europa.eu/health/patient_safety/policy_en

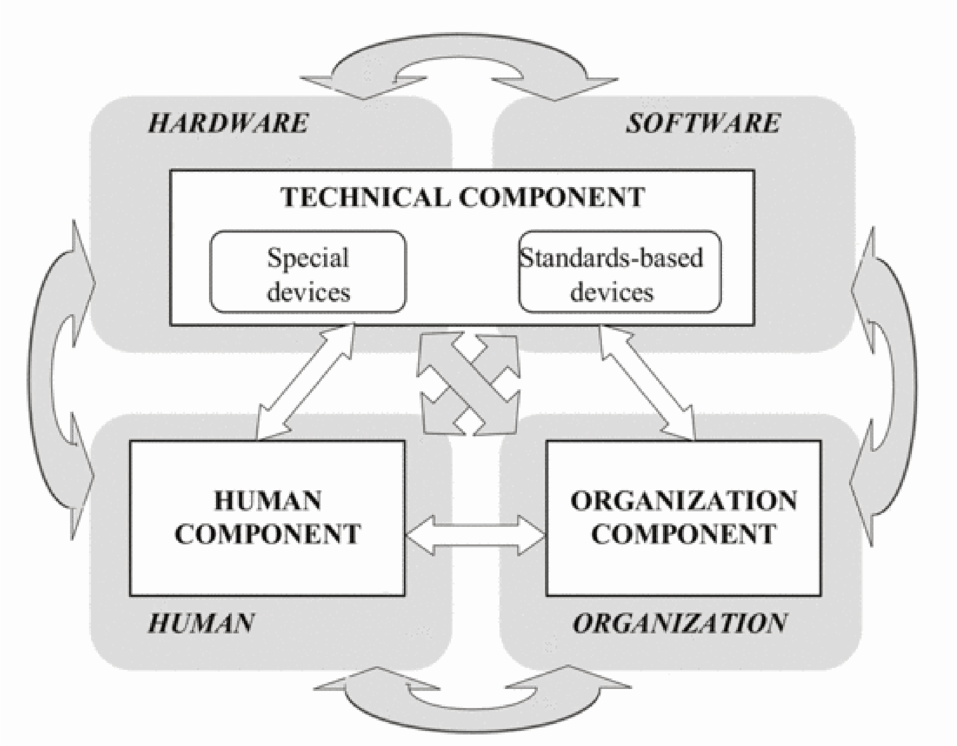
Safe vs Unsafe



- R. Amalberti, Y. Auroy, D. Berwick, and P. Barach, "Five system barriers to achieving ultrasafe health care," *Annals of Internal Medicine*, vol. 142, no. 9, p. 756, May 2005

Healthcare System

- Composed of many heterogeneous components.
- Problems of data collecting:
  - heterogeneous
  - uncertain (expert evaluation)
  - incompletely specified

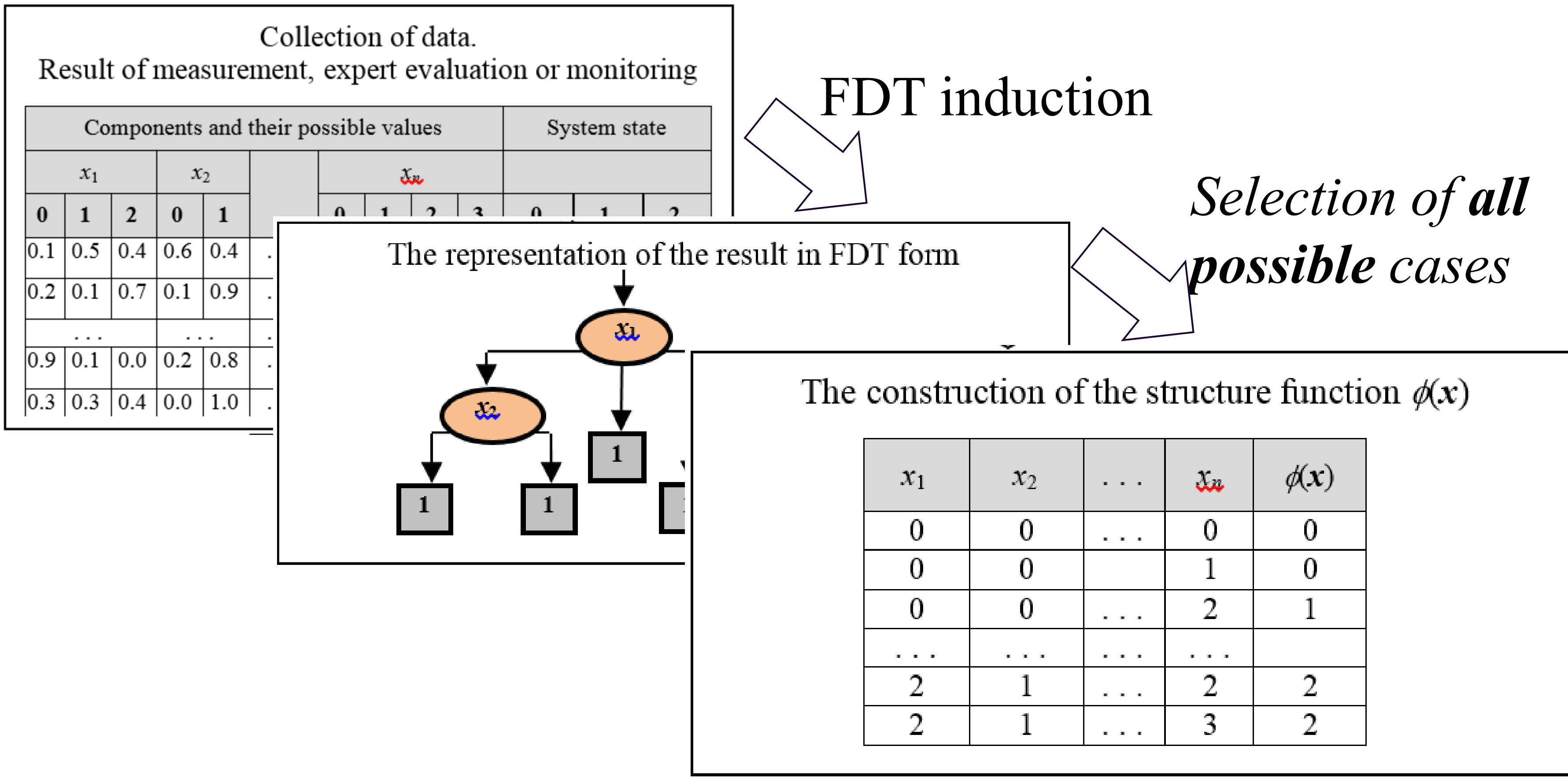


- E. Zaitseva, “Reliability analysis methods for healthcare system,” in *Human System Interactions (HSI), 2010 3rd Conference on*, 2010, pp. 211–216.

Reliability of Healthcare System

- Mainly qualitative approaches – focus on identification of steps that result in medical error.
- We try to develop a method for quantitative analysis. The method is a combination of tools of:
  - reliability analysis,
  - logic algebra,
  - data mining.

Reliability of Healthcare System



x_1	x_2	. . .	x_n	$\phi(x)$
0	0	. . .	0	0
0	0		1	0
0	0	. . .	2	1
. . .	. . .	. . .	. . .	
2	1	. . .	2	2
2	1	. . .	3	2

- V. Levashenko, E. Zaitseva, M. Kvassay, and T. M. Deserno, “Reliability estimation of healthcare systems using Fuzzy Decision Trees,” in *2016 Federated Conference on Computer Science and Information Systems (FedCSIS)*, 2016, pp. 331–340.

Reliability Analysis

Reliability Analysis

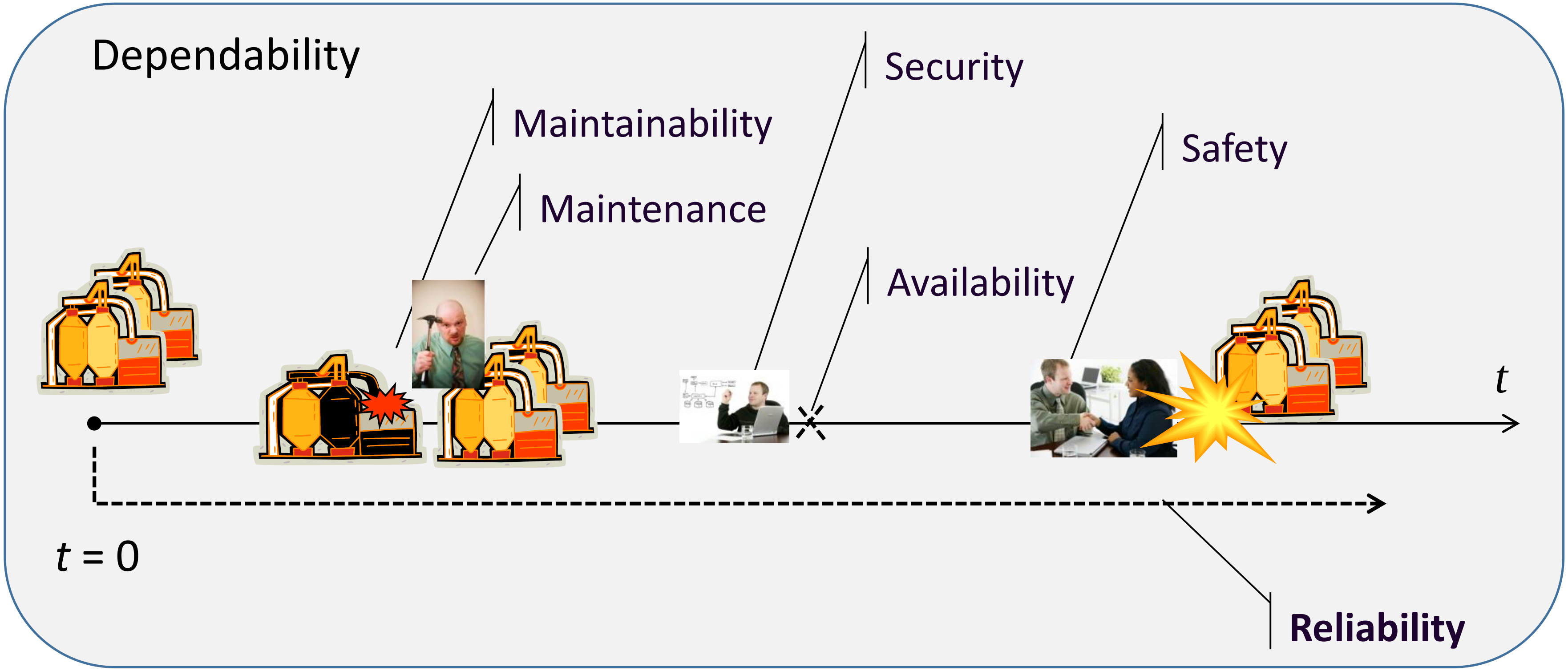
- Why systems fail?
- How to develop reliable systems?
- How to measure and test reliability in design, operation and management?
- How to maintain systems reliable, by maintenance, fault diagnosis and prognosis?

⇓

- How to model the system?
- How to quantify system reliability?
- How to represent, model and quantify

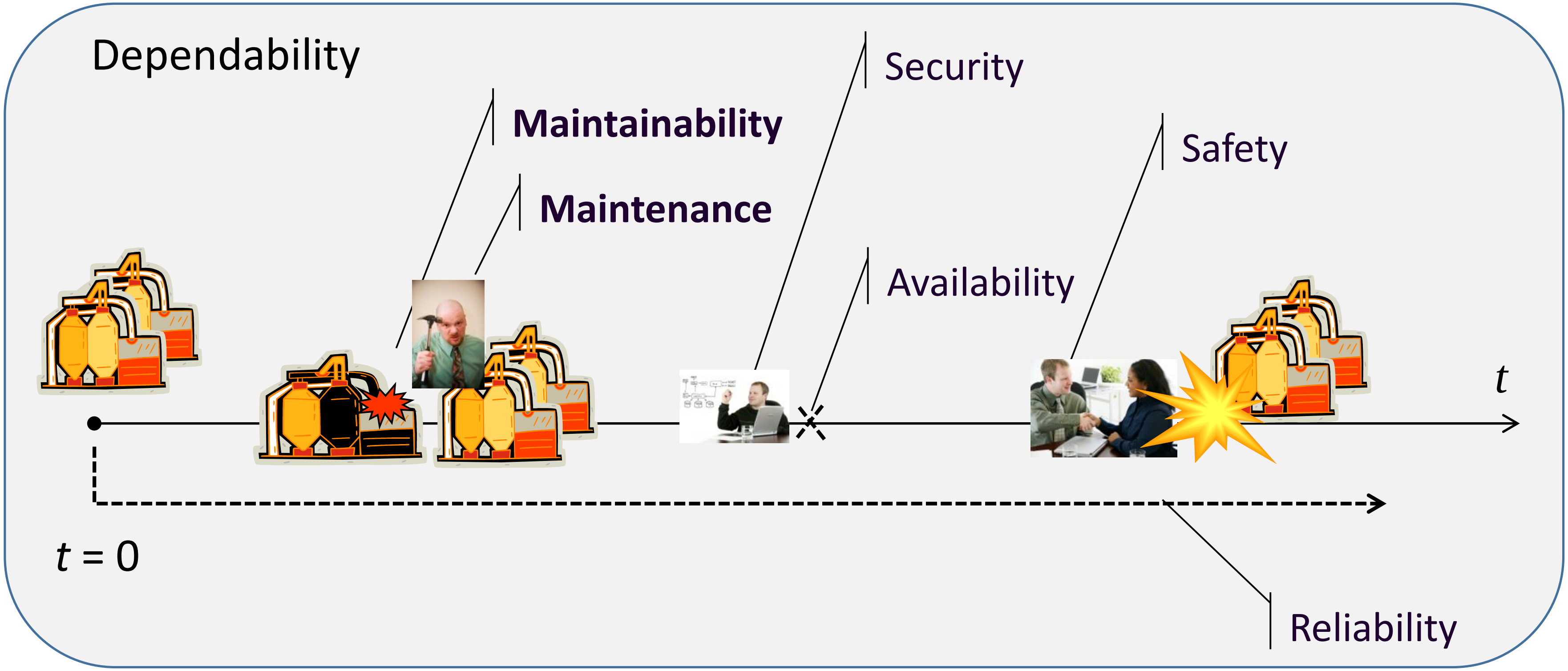
- E. Zio, “Reliability engineering: Old problems and new challenges,” *Reliability Engineering & System Safety*, vol. 94, no. 2, pp. 125–141, Feb. 2009.

Reliability – Basic Concepts



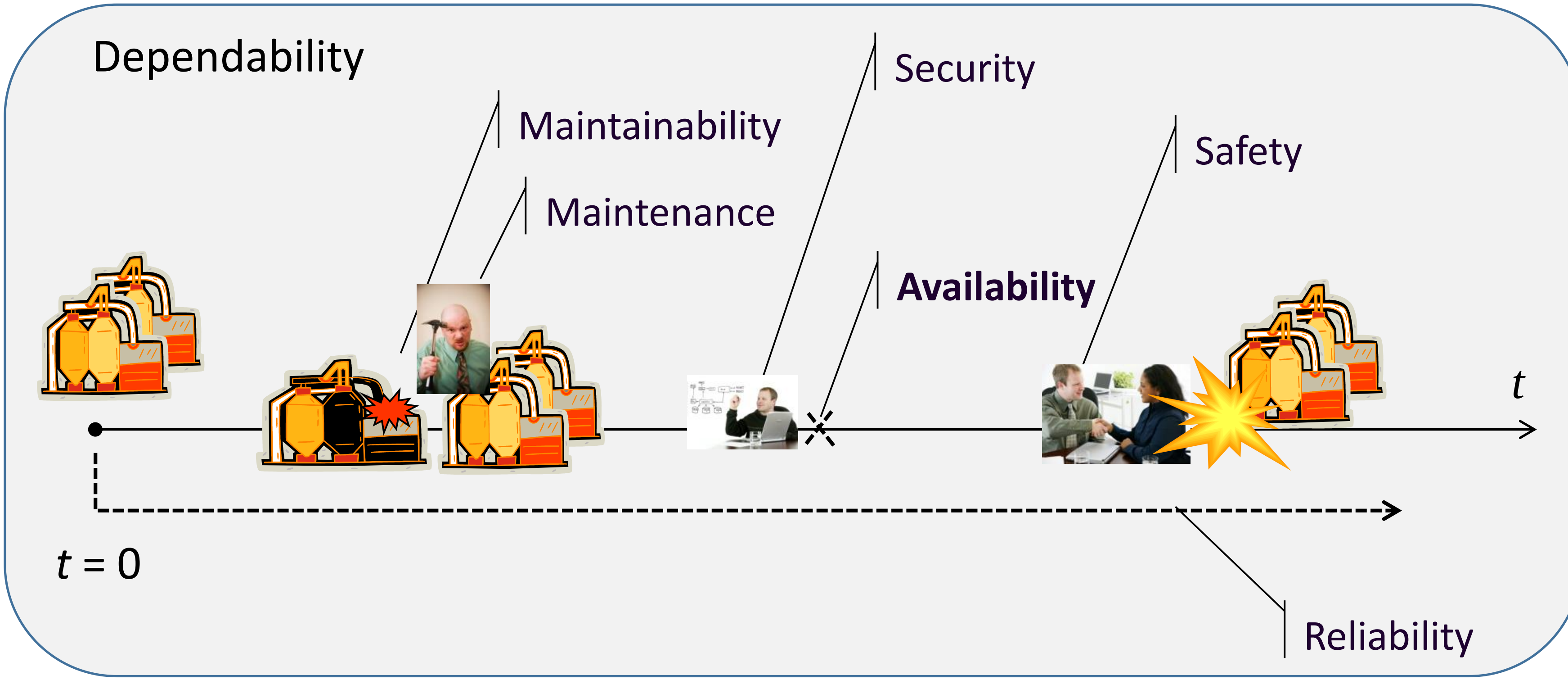
- **Reliability** – the probability that the system operates without failure in the interval <0, *t*>, given that it worked at time 0.

Reliability – Basic Concepts



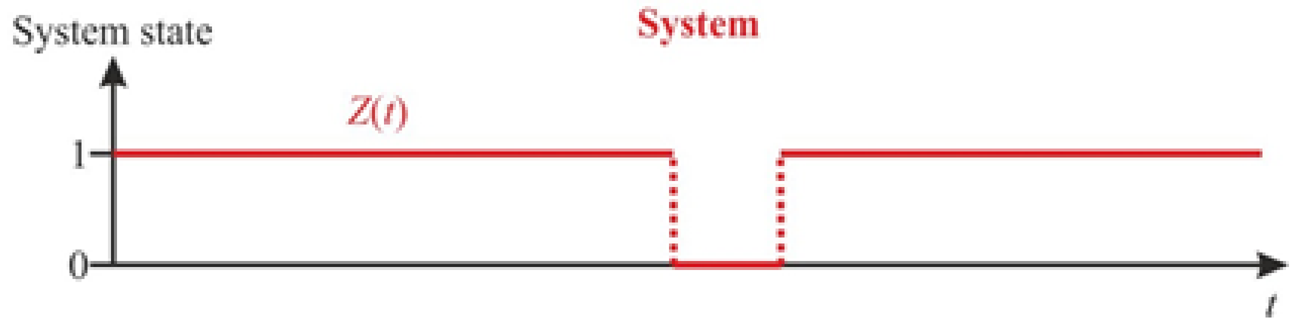
- **Maintainability** – the probability that the system will be repaired at time *t*, given that it failed at time 0.
- **Maintenance** – all actions that allows repairing system (corrective) or preventing its failure (preventive).

Reliability – Basic Concepts



- **Availability** – the probability that the system is functioning at time *t*.

Relationship between Reliability, Maintainability and Availability



Availability – complex characteristic, whose computation can be quite complicated, therefore:

- average interval availability:

$$A_{\text{avg}}(t) = \frac{1}{t}\int_0^t A(\tau)\,d\tau; \quad t > 0$$

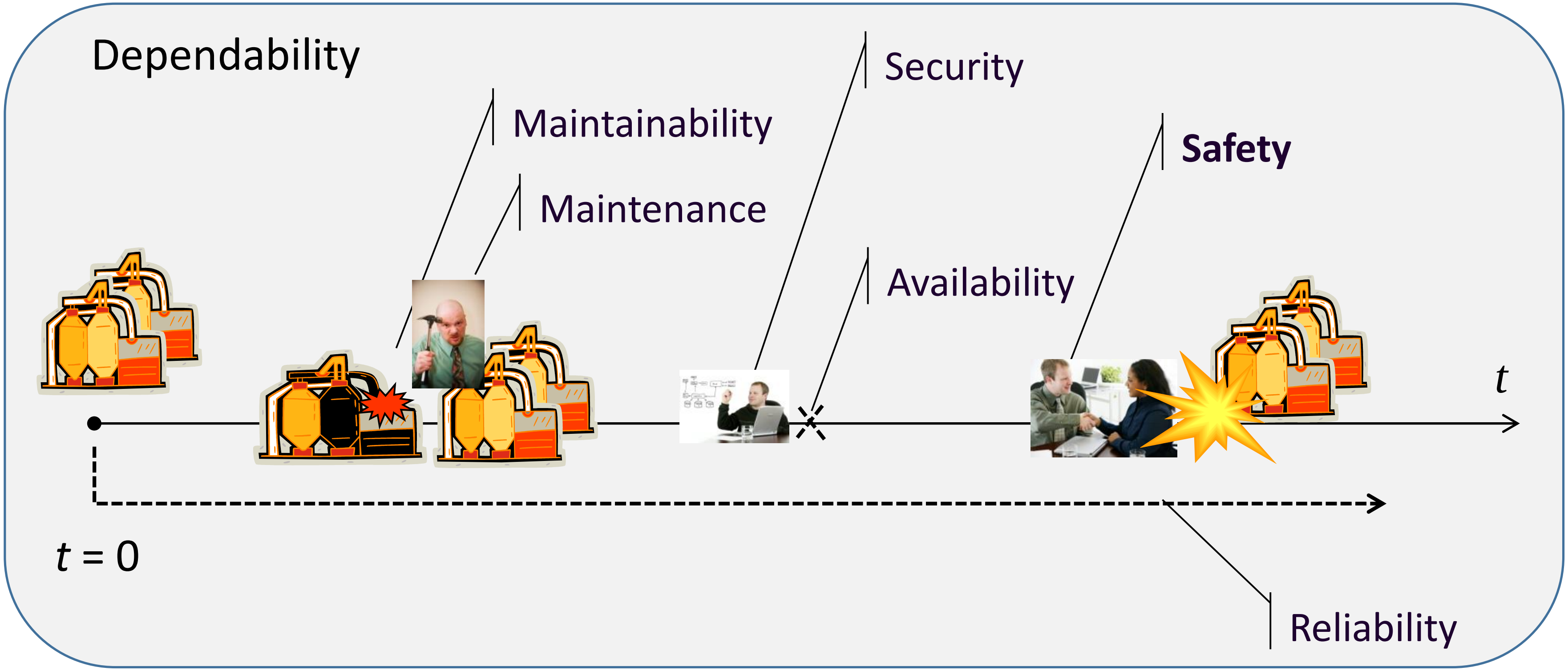
- average (steady-state) availability:

$$A_{\text{avg}} = \lim_{t\to\infty} A_{\text{avg}}(t) = \lim_{t\to\infty} A(t) = A$$

Reliability	Maintainability	Availability
If Constant	Increase ⬆	Increase ⬆
If Constant	Decrease ⬇	Decrease ⬇
Increase ⬆	If Constant	Increase ⬆
Decrease ⬇	If Constant	Decrease ⬇

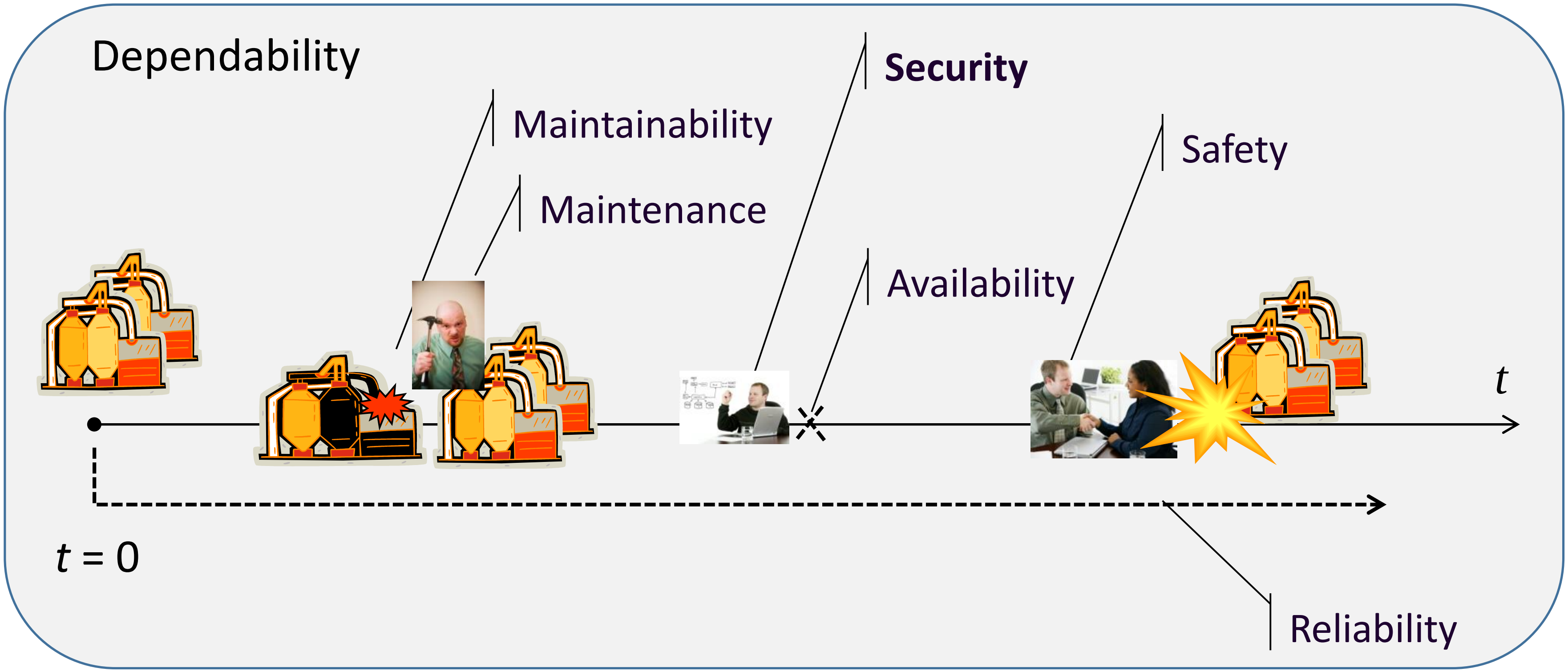
- M. Rausand and A. Høyland, *System Reliability Theory*, 2nd ed. Hoboken, NJ: John Wiley & Sons, Inc., 2004.

Reliability – Basic Concepts



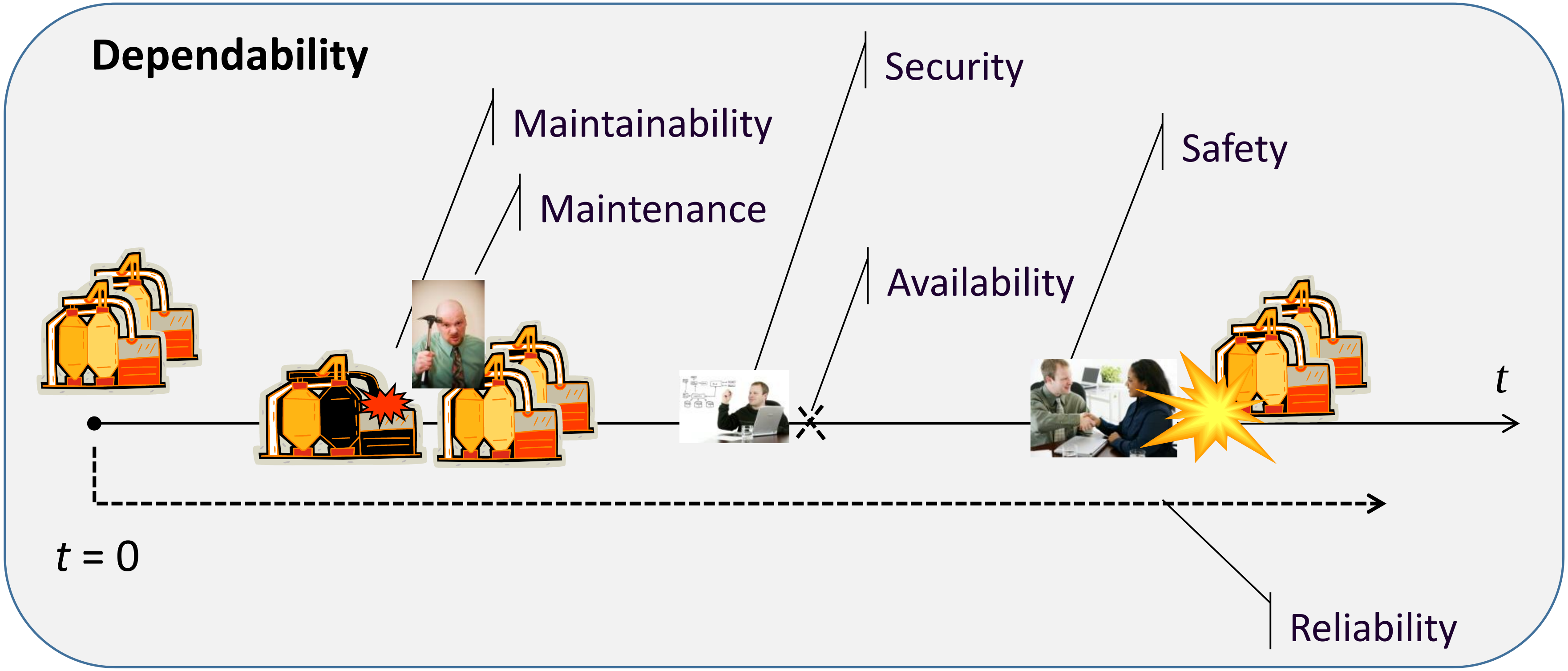
- **Safety** – the probability that the system will either perform its function correctly or will discontinue its operation in a safe way.

Reliability – Basic Concepts



- **Security** – the probability that the system is able to resist internal or external threats.

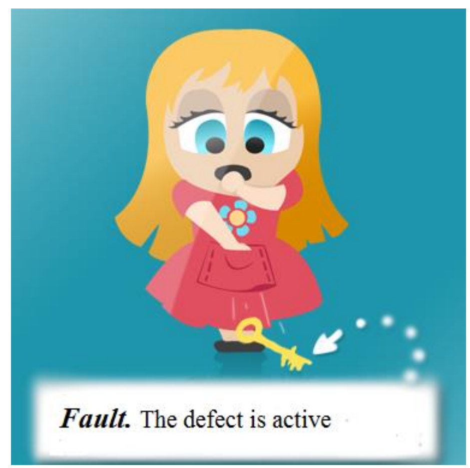
Reliability – Basic Concepts



- **Dependability** – the ability of the system to deliver its intended level of service to its users.

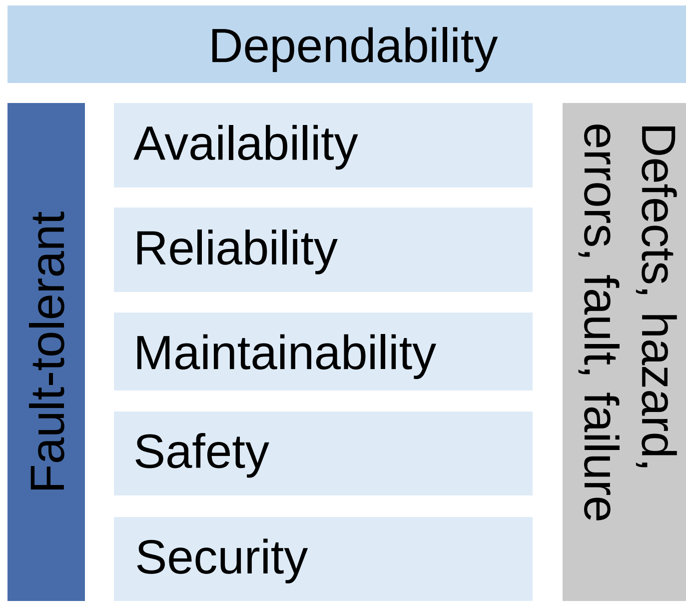
Cause and Effect Relationship

- **Error** – a deviation from correctness or accuracy.
- **Defect** – the departure of a quality characteristic from its specified value that results in a product not satisfying its normal usage requirements.
- **Fault** – a physical defect, imperfection or flaw that occurs in hardware or software.
- **Failure** – a non-performance of some action that is due or expected.



- A. Birolini, *Reliability Engineering*, 5th ed. Springer, 2007.

Reliability as Complex Problem

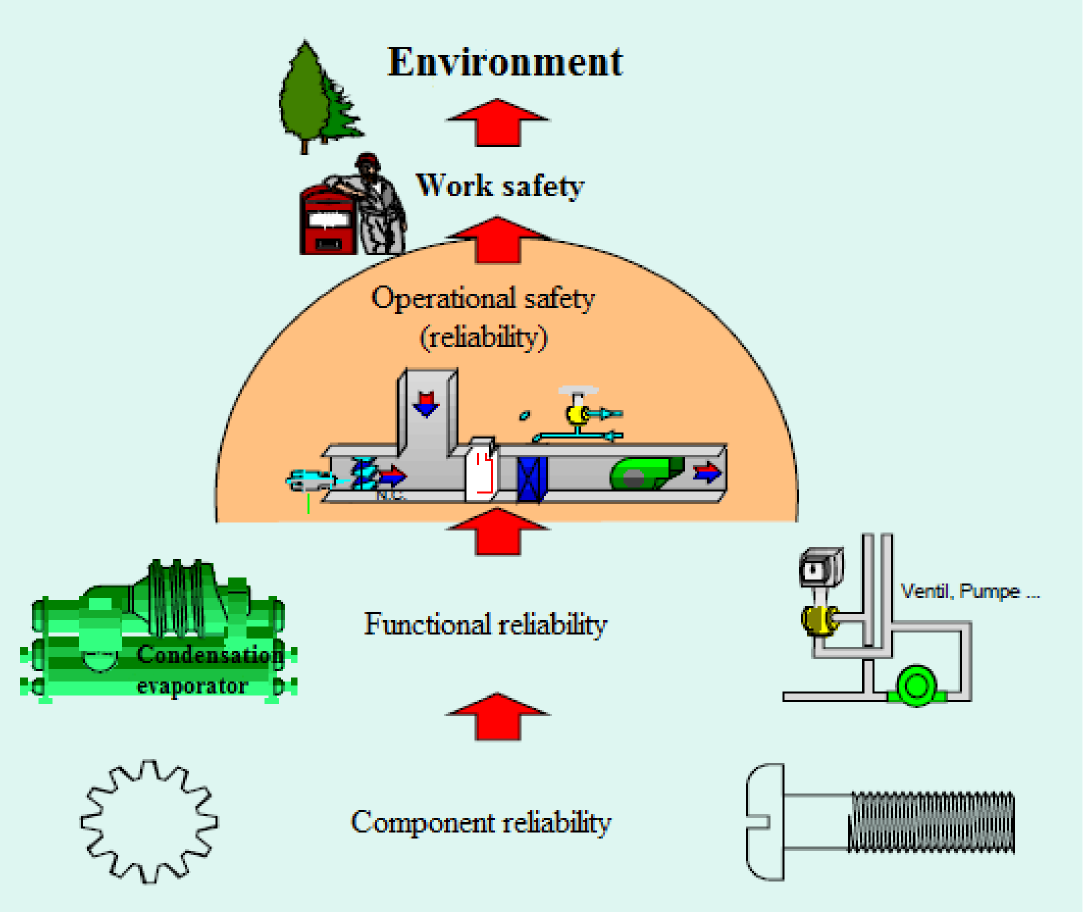


The main goal of reliability analysis is to increase the dependability/reliability of a system.

- A. Birolini, *Reliability Engineering*, 5th ed. Springer, 2007.

Different Views on Reliability

- Components create system.
- System is served by personnel.
- System and personnel interact with environment.



Two Approaches

- **Qualitative** – aims to identify, classify and rank the failure modes, or event combinations that would lead to system failures
- **Quantitative** – aims to evaluate in terms of probabilities the attributes of dependability (reliability, availability, safety)

Methods of Qualitative Analysis

- Checklist
- Preliminary hazard analysis
- Failure Mode and Effect Analysis (FMEA)
- Fault trees
- …

Preliminary Hazard Analysis

- **Hazard** – a situation with the potential for injury or fatality whereas **failure** is the actual event, be it hazardous or otherwise. The term **major hazard** is different only in degree and refers to certain large-scale potential incidents.
- **Preliminary hazard analysis** is a semi-quantitative analysis that is performed to:
  1. identify all potential hazards and accidental events that may lead to an accident;
  2. rank the identified accidental events according to their severity;
  3. identify required hazard controls and follow-up actions.

- The risk is established as a combination of a given event/consequence and a severity of the same event/consequence. This will enable a ranking of the events/consequences in a risk matrix:

Frequency/ consequence	1 Very unlikely	2 Remote	3 Occasional	4 Probable	5 Frequent
Catastrophic	Yellow	Red	Red	Red	Red
Critical	Green	Yellow	Yellow	Red	Red
Major	Green	Green	Yellow	Yellow	Red
Minor	Green	Green	Green	Yellow	Yellow

Green: Acceptable - only ALARP actions considered

Yellow: Acceptable - use ALARP principle and consider further investigations

Red: Not acceptable - risk reducing measures required

- E. Zio, *An Introduction to the Basic of Reliability and Risk Analysis*. London, UK: World Scientific, 2007.

Failure Mode and Effect Analysis (FMEA)

- **FMEA** is a systematic procedure for identifying the modes of failures and for evaluating their consequences. It is a tabular procedure which considers hazards in terms of single-event chains and their consequences.
- It is a qualitative method, of inductive nature, which aims at identifying those failure modes of the components which could disable system operation or become initiators of accidents with significant external consequences.
- The basic questions which must be answered by the analyst are:
  - How can each component or subsystem fail? (What is the **failure mode**?)
  - What cause might produce this failure? (What is the **failure mechanism**?)
  - What are the **effects** of each failure if it does occur?

- E. Zio, *An Introduction to the Basic of Reliability and Risk Analysis*. London, UK: World Scientific, 2007.

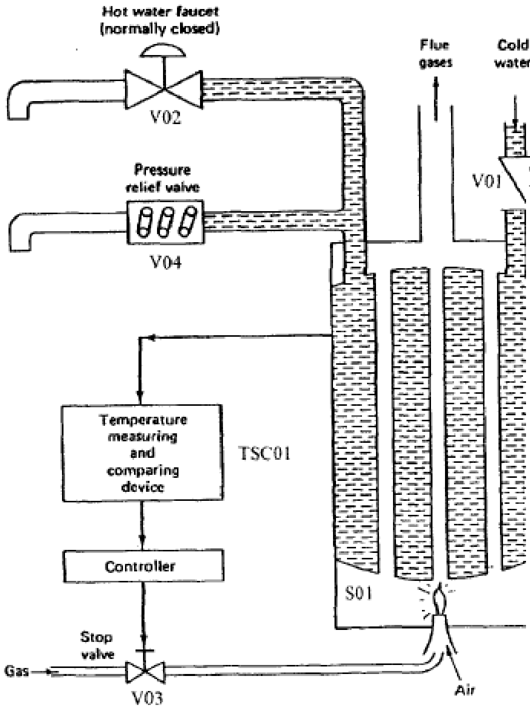
Failure Mode and Effect Analysis

- Once the FMEA is completed, it assists the analyst in:
  - selecting, during initial stages, various design alternatives with high reliability and high safety potential;
  - ensuring that all possible failure modes, and their effects on operational success of the system, have been taken into account;
  - identifying potential failures and the magnitude of their effects on the system;
  - developing testing and checkout methods.

SYSTEM:									
OPERATION MODE:									
Component	**Failure mode**	**Effects on other components**	**Effects on subsystem**	**Effects on plant**	**Probability**	**Criticality**	**Detection methods**	**Protections and mitigation**	**Remarks**
Description	Failure modes relevant for the operational mode indicated	Effects of failure mode on adjacent components and surrounding environment	Effects on the functionality of the subsystem	Effects on the functionality and availability of the entire plant	Probability of failure occurrence (usually qualitative)	Criticality rank of the failure mode on the basis of its effects and probability (qualitative estimation of risk)	Methods of detection of the occurrence of the failure event	Protections and measures to avoid the failure occurrence	Remarks and suggestions on the need to consider the failure mode as accident initiator

- E. Zio, *An Introduction to the Basic of Reliability and Risk Analysis*. London, UK: World Scientific, 2007.

Failure Mode and Effect Analysis – Domestic Hot Water System

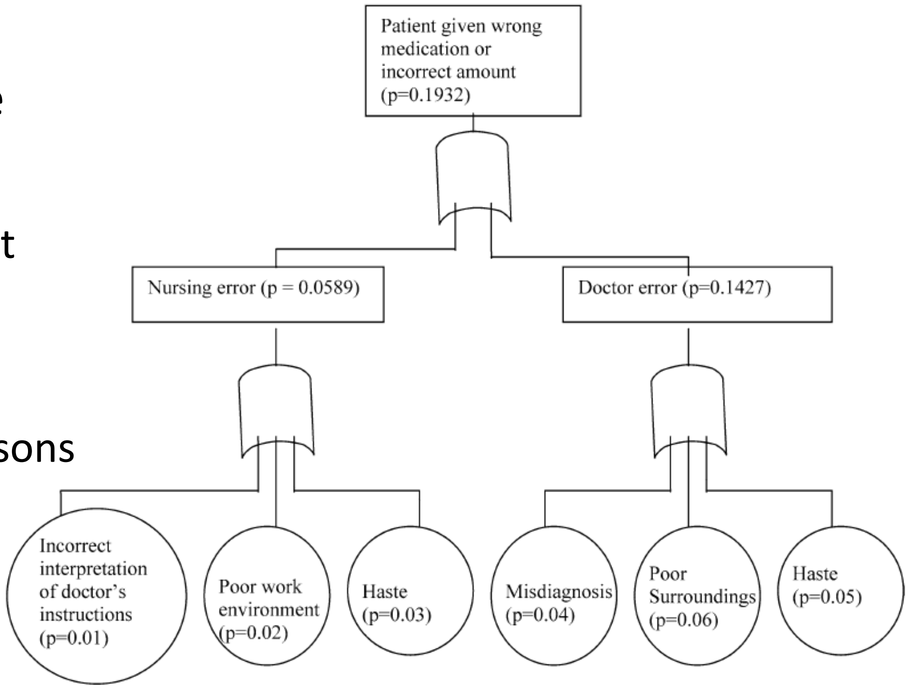


Component	*Failure mode*	*Effects on whole system*	*Critically class*	*Failure frequency*	*Detection methods*	*Compensating provision and remarks*
Pressure relief valve (V04)	*Jammed open*	*Increasing operation of temperature sensing controller; Gas flow due to hot water loss*	*Safe*	*Reasonably probable*	*Observe at pressure relief valve*	*Shut off water supply, reseal or replace relief valve*
	Jammed close	*Rupture of container or pipes*	*Critical*	*Probable*	*Manual testing*	*If combined with other component failures, otherwise this failure has no consequence*
Gas valve (V03)	*Jammed open*	*Burner continues to operate, pressure relief valve opens*	*Critical*	*Reasonably probable*	*Water at faucet too hot: pressure relief valve open (observation)*	*Open hot water faucet to relieve pressure. Shut off gas supply. Pressure relief valve compensates. IE1.*
	Jammed close	*Burner ceases to operate*	*Safe*	*Remote*	*Observe at output (Water temperature too low)*	
Temperature measuring and comparing device (Tsc01)	*Fail to react to temperature rise above preset level*	*Controller, gas valve, burner continue to function "on". Pressure relief valve opens*	*Critical*	*Remote*	*Observe at output (faucet)*	*Pressure relief valve compensates. Open hot water faucet to relieve pressure. Shut off gas supply. IE2.*
	Fail to react to temperature drop below preset level	*Controller, gas valve, burner continue to function "off".*	*Safe*	*Remote*	*Observe at output (faucet)*	
IE: initiating Event						

- E. Zio, *An Introduction to the Basic of Reliability and Risk Analysis*. London, UK: World Scientific, 2007.

Fault Trees

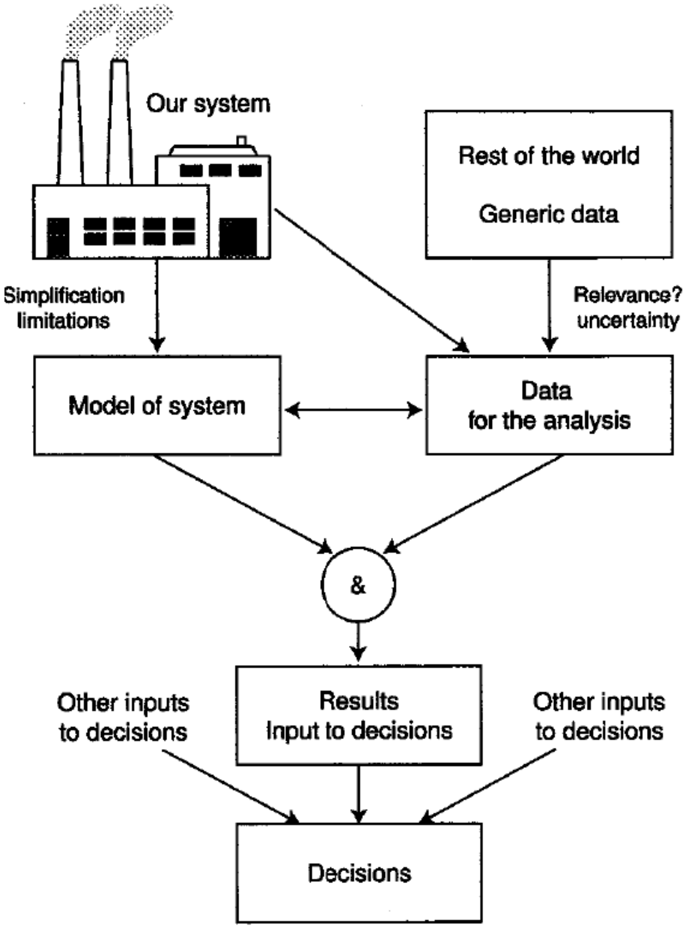
- Fault trees represent hierarchical approach.
- They are useful for both qualitative and quantitative analyses because they:
  - force the analyst to actively seek out failure events (success events) in a deductive manner;
  - provide a visual display of how the system can fail, and thus aid understanding of the system by persons other than the designer;
  - point out critical aspects of systems failure (system success);
  - provide a systematic basis for quantitative analysis of reliability.
- The analysis based on fault trees is performed by identification so-called minimal cut sets.



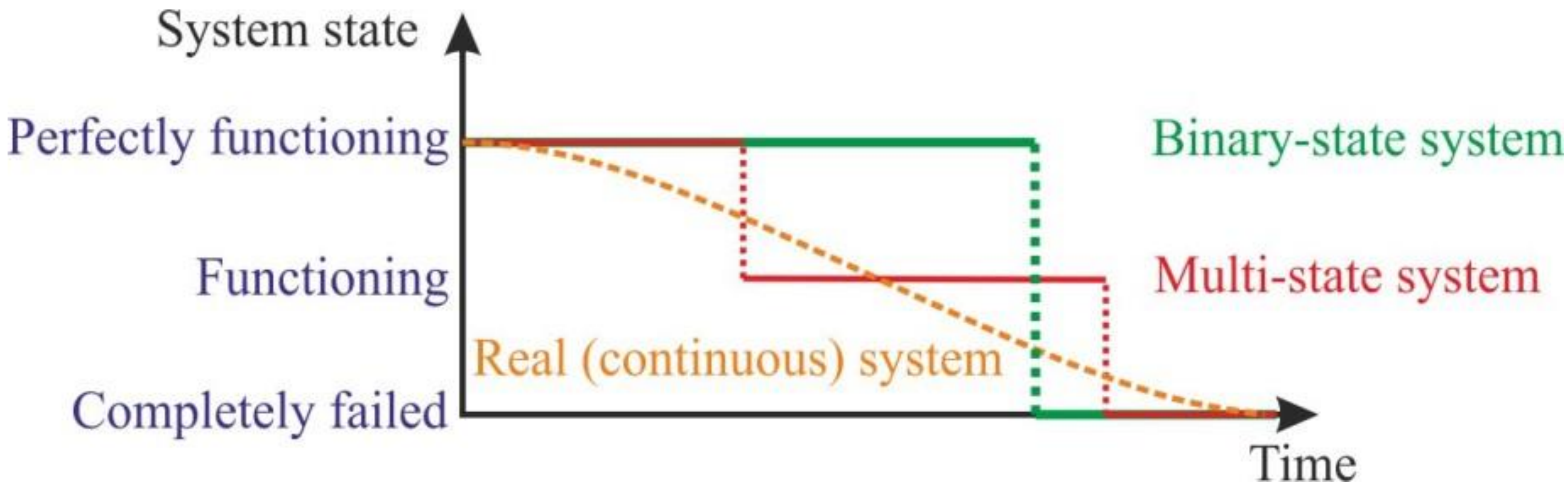
- B. S. Dhillon, *Human Reliability and Error in Medicine*. Singapore, SG: World Scientific, 2003.
- E. Zio, *An Introduction to the Basic of Reliability and Risk Analysis*. London, UK: World Scientific, 2007.

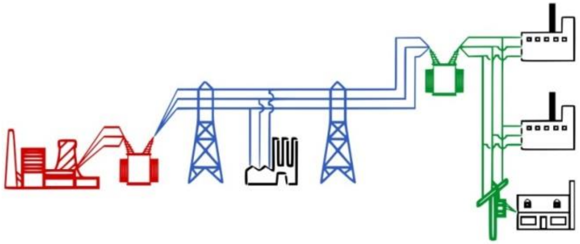
Quantitative Analysis

- Principal steps for reliability estimation of complex systems:
  1. definition of number of performance levels for the system model;
  2. mathematical representation of the system model;
  3. quantification of the system model (calculation of indices and measures, for example importance measures);
  4. measuring behavior of the system.



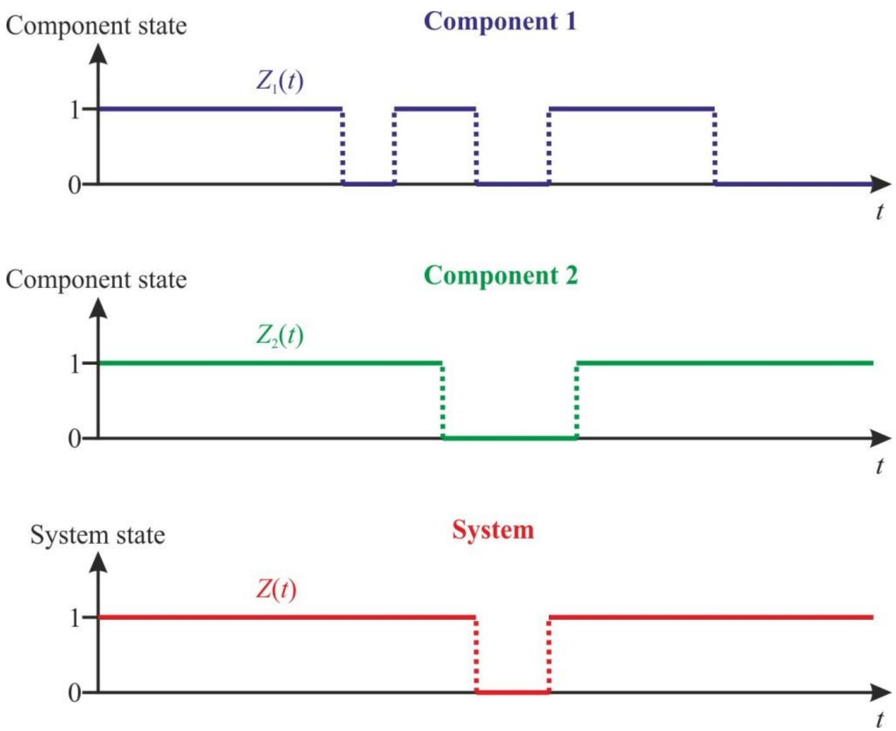
Number of Performance Levels

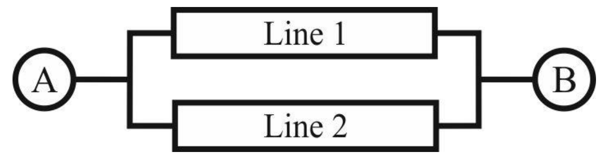


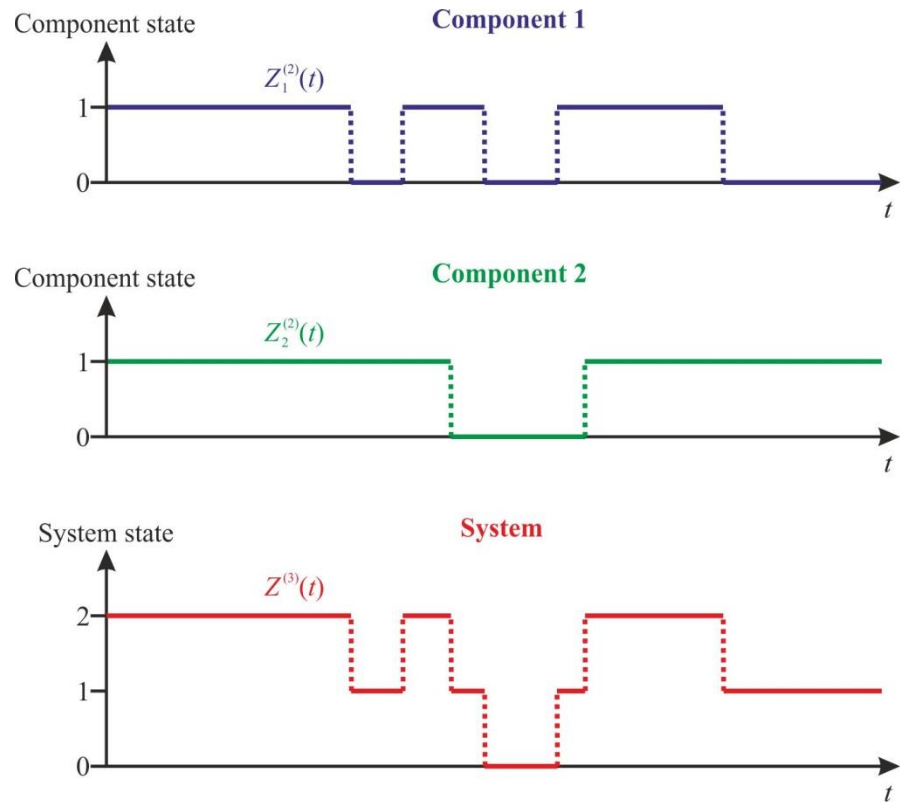


Binary- and Multi-State Systems

RAID 1







Number of Performance Levels

Properties	BSS	MSS
Exactness	–	+
Computational complexity	+	–
Elaboration	+	–

Principal problems for MSS application:

- *High Dimension of the MSS :* $\prod_{i=1}^{n} m_i$
- *Elaboration of new algorithms, methods and indices.*

Mathematical Representation

Markov & semi-Markov model

Universal generating function

Monte-Carlo simulation model

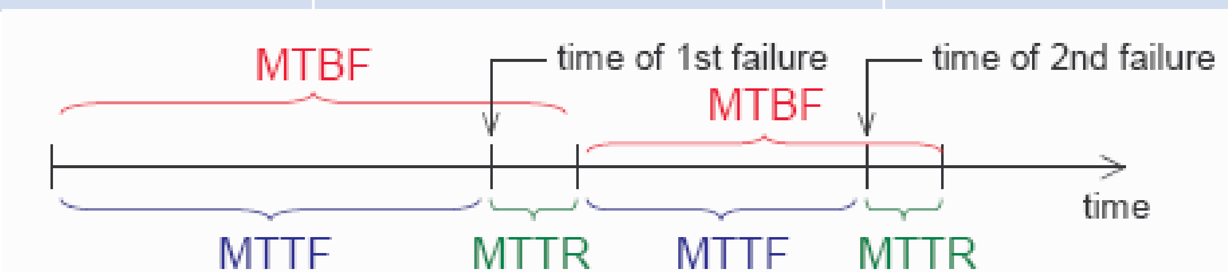
Structure function

Structure function based methods for reliability analysis:

- Fault Trees analysis
- Reliability Block Diagram analysis
- Minimal Cut/Path set based methods

Estimation of Common Measures

Non-repairable systems	Repairable systems	
Reliability $R(t)=1-\dfrac{\text{number of faults which is detected}}{\text{total number of items}}$		**Availability** $A(t)=\dfrac{\text{number of working items}}{\text{number of items}}$
Failure rate $\lambda(t)=\dfrac{\text{number of failed items}}{\text{number of working items in time } t_0}$	**Repair rate** $\mu(t)=\dfrac{\text{number of restored items}}{\text{number of failure items in time } t_0}$	
Mean time to failure $MTTF=\dfrac{1}{N}\cdot\sum_{i=1}^{N}t_i$	**Mean time to repair** $MTTR=\dfrac{1}{N}\cdot\sum_{i=1}^{N}t_i$	**Mean time between failures** $MTBF=MTTF+MTTR$



- M. Rausand and A. Høyland, *System Reliability Theory*, 2nd ed. Hoboken, NJ: John Wiley & Sons, Inc., 2004.

Example

Time	$L(t)$	$R(t)$	$F(t)$	$\lambda(t)$
0	1 023 102	1.00000	0.00000	0.02258
1	1 000 000	0.97742	0.02258	0.00577
2	994 230	0.97178	0.02822	0.00414
3	990 114	0.96776	0.03224	0.00338
4	986 767	0.96449	0.03551	0.00299
5	983 817	0.96160	0.03840	0.01221
10	971 804	0.94986	0.05014	0.00981
15	962 270	0.94054	0.05946	0.01121
20	951 483	0.93000	0.07000	0.01291
25	939 197	0.91799	0.08201	0.01553
30	924 609	0.90373	0.09627	0.01953
35	906 554	0.88608	0.11392	0.02560
40	883 342	0.86340	0.13660	0.03485
45	852 554	0.83330	0.16670	0.04886
50	810 900	0.79259	0.20741	0.06993
55	754 191	0.73716	0.26284	0.10133
60	677 771	0.66247	0.33753	0.14738
65	577 882	0.56483	0.43517	0.21342
70	454 548	0.44428	0.55572	0.30484
75	315 982	0.30885	0.69115	0.42476
80	181 765	0.17766	0.82234	0.56966
85	78 221	0.07645	0.92355	0.72415
90	21 577	0.02109	0.97891	0.86045

Measures for item in time $t = 75$:

- $R(t) = 0.3088$
- $F(t) = 0.6912$
- $\lambda(t) = 0.4248$

Mean time to failure:

- $MTTF = 62.7373$

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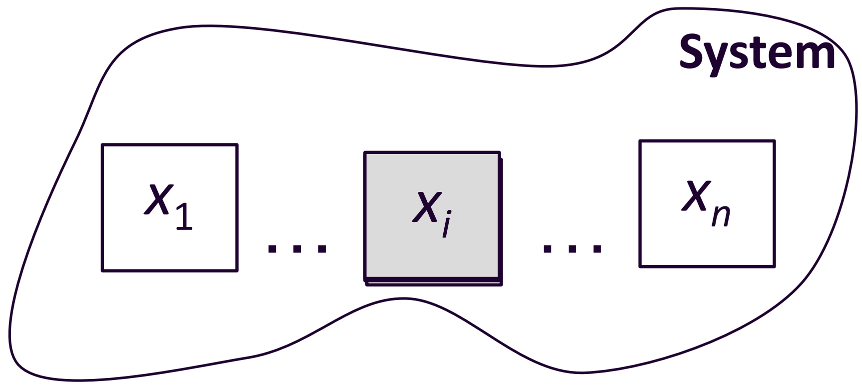
Boolean Functions and Binary-State Systems

Binary-State System



- n – number of system components
- for i = 1,2,…, n:
  - x_i – state of component i
    - 0 – component is failed
    - 1 – component is functioning
  - p_i – probability that the i-th component is working
  - q_i – probability of failure of the i-th component

- M. Rausand and A. Høyland, *System Reliability Theory*, 2nd ed. Hoboken, NJ: John Wiley & Sons, Inc., 2004.

Structure Function

- Structure function defines system topology:

$$\phi(x_1, x_2, \ldots, x_n) = \phi(\boldsymbol{x}): \{0, 1\}^n \rightarrow \{0, 1\}$$

Structure function
- state of the system at a fixed time
- state of component *i* at the fixed time

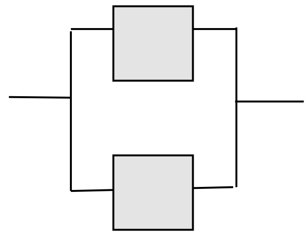
Boolean function
- function value
- value of the *i*-th variable of the function

Tools of Boolean algebra can be used in reliability analysis of binary-state systems.

Positive and Negative Logic

- **Positive logic**
  - Question: When is system functioning?
  - Methods: Reliability block diagram, Minimal path sets
  - Variable: x_i
  - Parameters: Survival probabilities, Availability
- **Negative logic**
  - Question: When does system fail?
  - Methods: Fault tree, Minimal cut sets
  - Variable: $\neg x_i$
  - Parameters: Failure probabilities, Unavailability

Representation of Structure Function



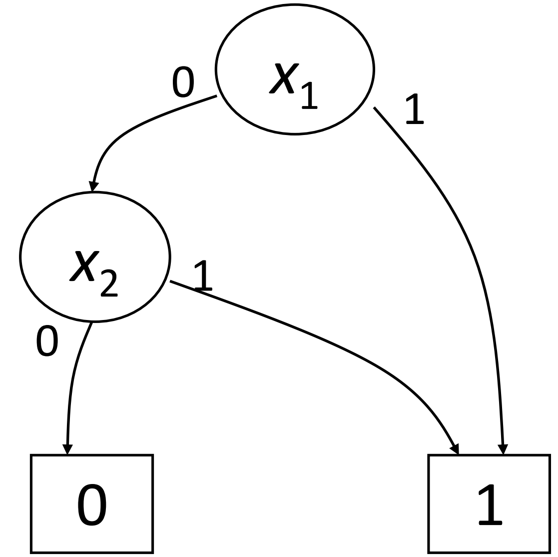
Analytical Description (formula)

$\phi(\boldsymbol{x}) = \mathrm{OR}(x_1, x_2)$

Truth Table

x_1 x_2	$\phi(\boldsymbol{x})$
0 0	**0**
0 1	**1**
1 0	**1**
1 1	**1**

Binary Decision Diagram



What are pros and cons of these approaches?

Structure Function – Analytical Description

- Logical representation:

$$\phi(\boldsymbol{x}) = \mathrm{OR}(x_1, x_2)$$

- Arithmetical representation:

$$\phi(\boldsymbol{x}) = x_1 + x_2 - x_1 x_2$$

- Logical-probabilistic representation:

$$A(\boldsymbol{p}) = p_1 + p_2 - p_1 p_2$$

Structure Function – Analytical Description

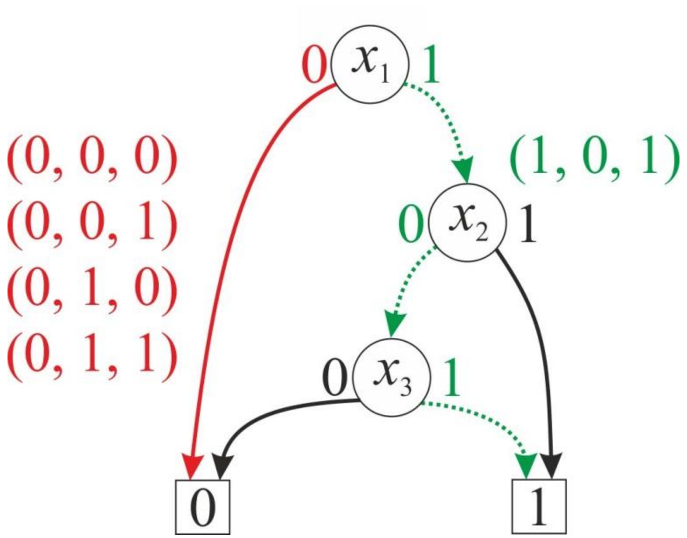
- Transform the following logical functions into arithmetical functions:
  - $f_1(\boldsymbol{x}) = \mathrm{NOT}(x_1)$
  - $f_2(\boldsymbol{x}) = \mathrm{AND}(x_1, x_2)$
  - $f_3(\boldsymbol{x}) = \mathrm{OR}(x_1, x_2, x_3)$
  - $f_4(\boldsymbol{x}) = \mathrm{OR}(x_1, \mathrm{AND}(x_2, x_3))$
  - $f_5(\boldsymbol{x}) = \mathrm{NOT}(\mathrm{XOR}(x_1, \mathrm{AND}(x_2, x_3)))$

Structure Function – Binary Decision Diagram

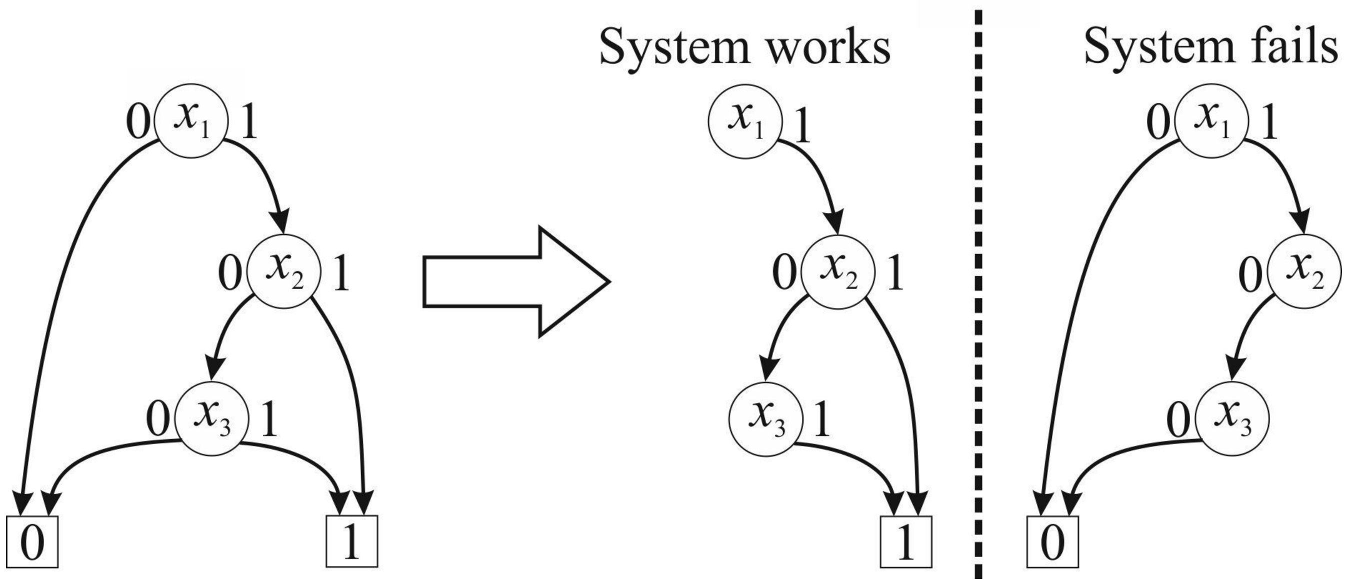
Truth Table

x_1	x_2	x_3	$\phi(\boldsymbol{x})$
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

Binary Decision Diagram



Structure Function – Computations based on Binary Decision Diagram

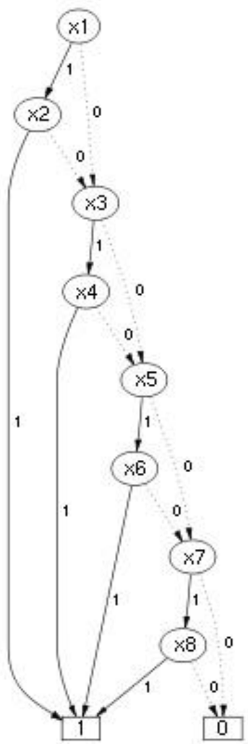


$A = p_1(q_2p_3 + p_2)$ $\qquad$ $U = q_1 + p_1q_2q_3$

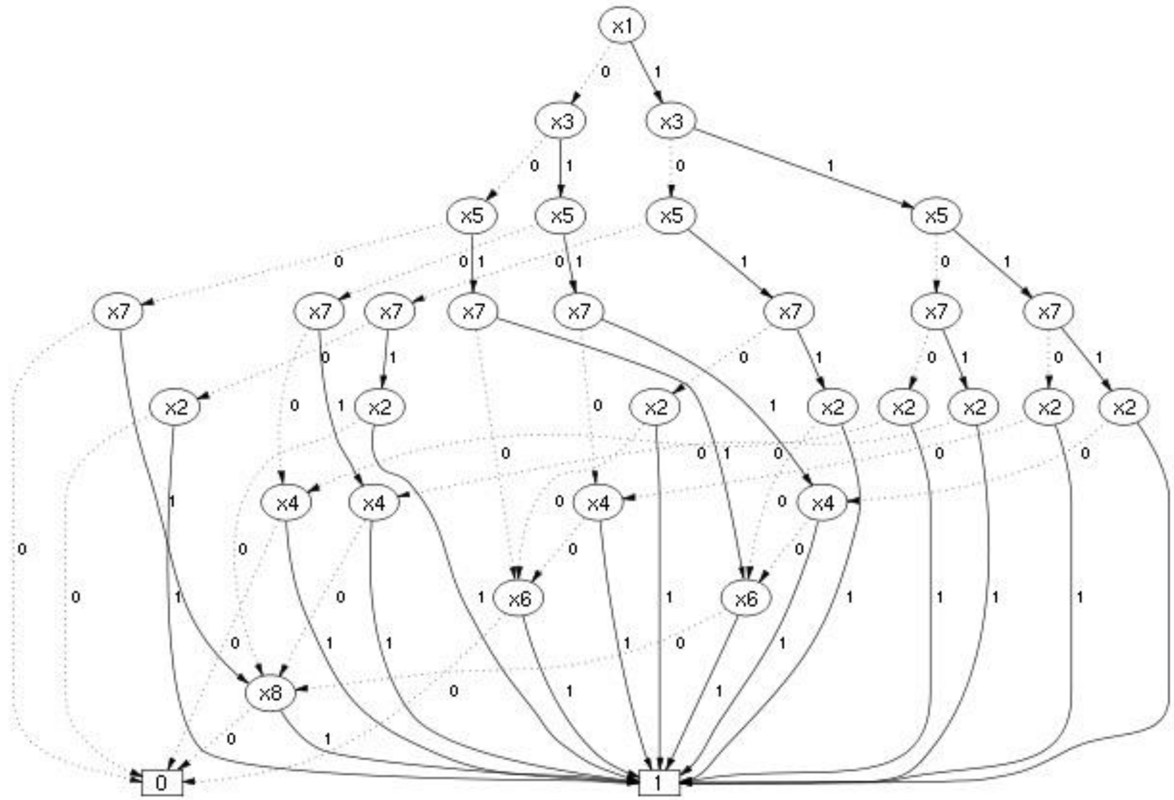
- **Try to express the following function in the form of BDD**:
  - $f(\boldsymbol{x})$ = OR(AND(x_1,x_3), AND(x_2,x_4))

Structure Function – Problems of Binary Decision Diagrams

Good



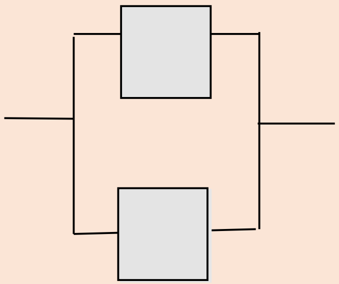
Bad



- Wikipedia: Binary Decision Diagram (https://en.wikipedia.org/wiki/Binary_decision_diagram)

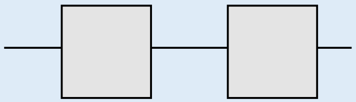
Reliability Block Diagrams and Typical Structures

Parallel



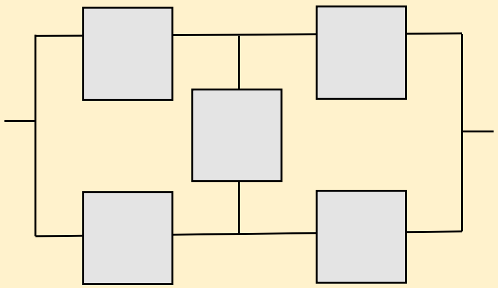
x_1 x_2	$\phi(\boldsymbol{x})$
0 0	**0**
0 1	**1**
1 0	**1**
1 1	**1**

Series



x_1 x_2	$\phi(\boldsymbol{x})$
0 0	**0**
0 1	**0**
1 0	**0**
1 1	**1**

Bridge



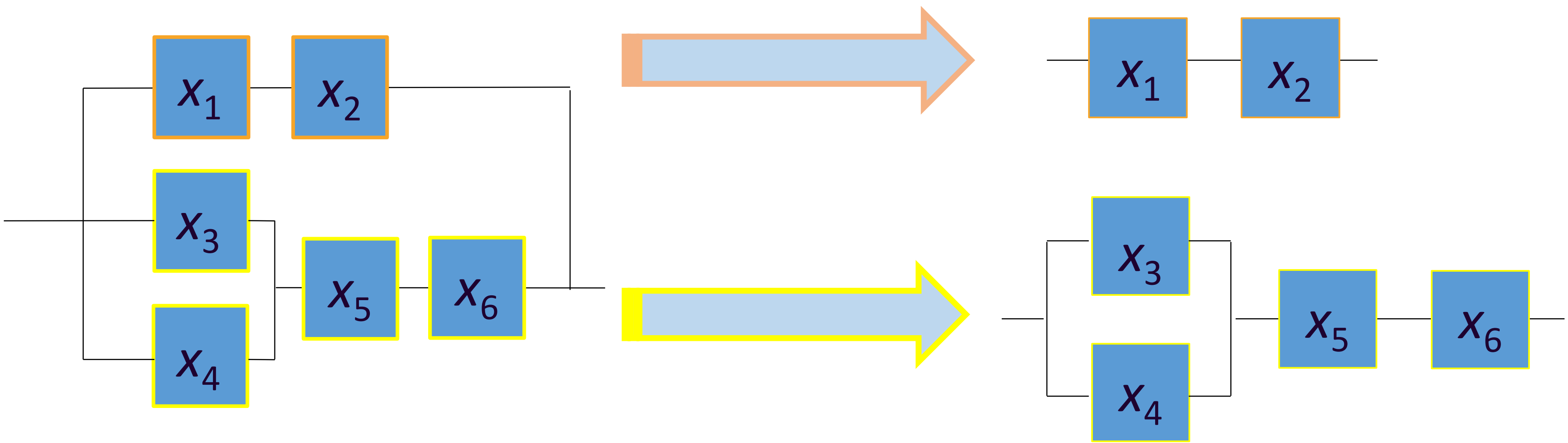
How is the structure function defined?

k-out-of-*n*

x_1 x_2 x_3	$\phi(\boldsymbol{x})$
0 0 0	**0**
0 0 1	**0**
0 1 0	**0**
1 0 0	**0**
1 0 1	**1**
1 1 0	**1**
1 1 1	**1**

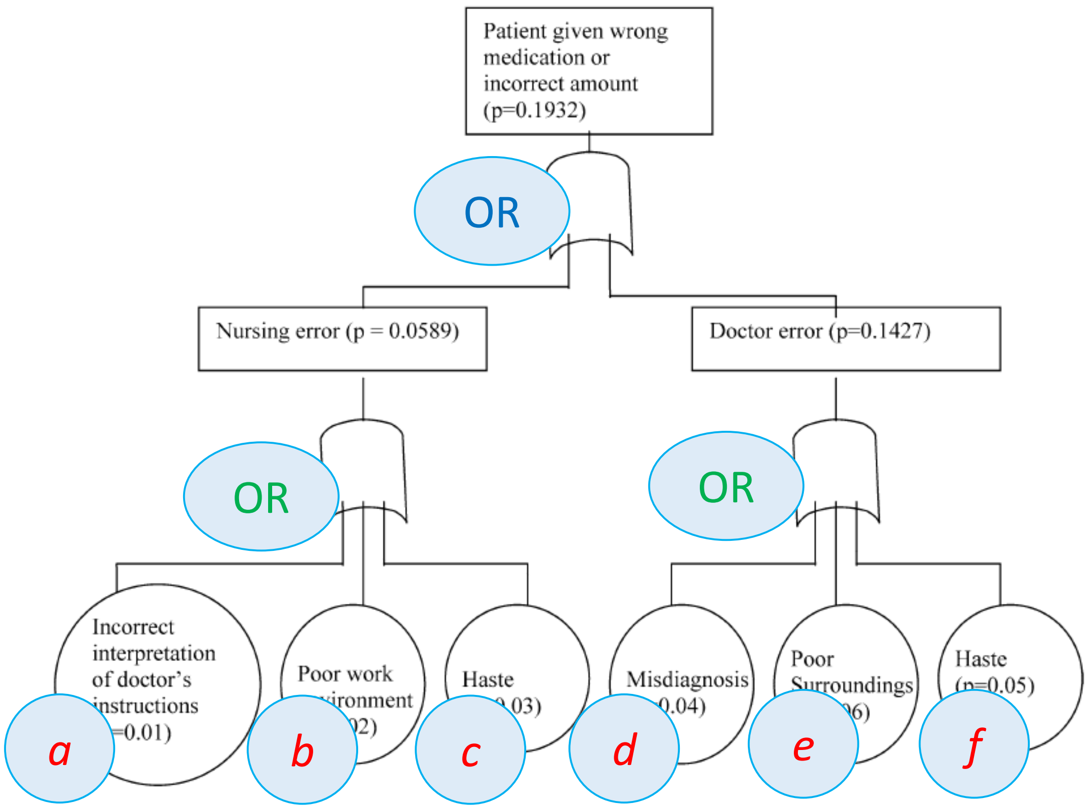
How does RBD look like?

Reliability Block Diagrams



$$\phi(\boldsymbol{x}) = ?$$
$$A(\boldsymbol{p}) = ?$$

Fault Trees (again)



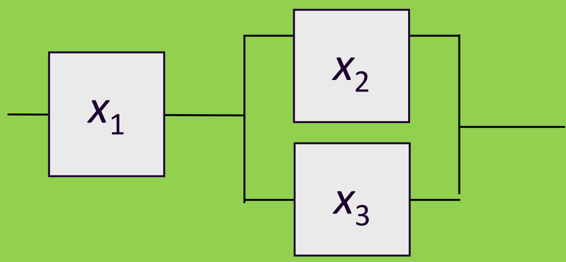
OR(OR(*a*,*b*,*c*), OR(*d*,*e*,*f*))

- **How do we obtain availability and unavailability?**
- **Can we transform the fault tree into reliability block diagram?**

- B. S. Dhillon, *Human Reliability and Error in Medicine*. Singapore, SG: World Scientific, 2003.

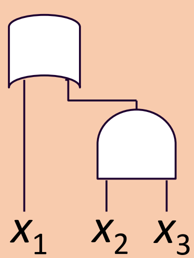
Reliability Block Diagram and Fault Tree

- **Positive logic**
  - Question: When is system functioning?



$$U \Rightarrow x_1 \wedge (x_2 \vee x_3)$$

- **Negative logic**
  - Question: When does system fail?



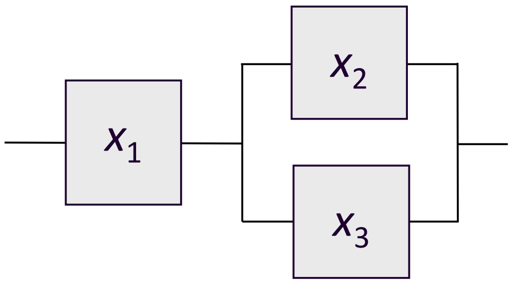
$$U \Rightarrow \bar{x}_1 \vee (\bar{x}_2 \wedge \bar{x}_3)$$

Minimal Cut Set

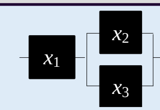
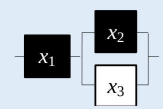
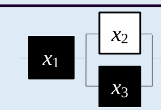
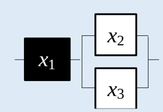
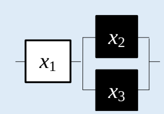
- Reliability block diagram:
  - A minimal set of components whose failure results in system failure.
- Fault tree:
  - A minimal set of events whose occurrence causes occurrence of the top event.
- Identification of the minimal cut sets is equivalent to finding all the prime implicates of the structure function.
- The dual concept is known as **Minimal Path Set**. The minimal path sets of the system agree with the prime implicants of the function.

- M. Rausand and A. Høyland, *System Reliability Theory*, 2nd ed. Hoboken, NJ: John Wiley & Sons, Inc., 2004.

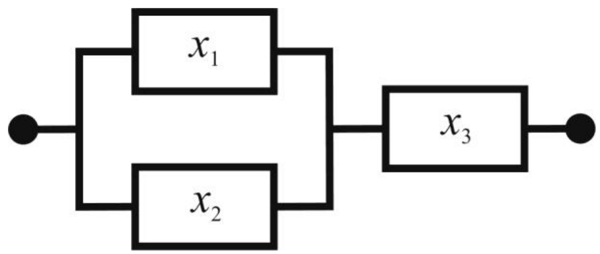
Minimal Cut Sets



x_1	x_2	x_3	$\phi(\boldsymbol{x})$
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

x_1	x_2	x_3	$\phi(\boldsymbol{x})$	RBD	Cut	Minimal cut
0	0	0	**0**		$\{c_1 c_2 c_3\}$	no
0	0	1	**0**		$\{c_1 c_2\}$	no
0	1	0	**0**		$\{c_1 c_3\}$	no
0	1	1	**0**		$\{c_1\}$	yes
1	0	0	**0**		$\{c_2 c_3\}$	yes

Importance Analysis



$\phi(x_1, x_2, x_3)$		x_3	
x_1	x_2	0	1
0	0	0	0
0	1	0	1
1	0	0	1
1	1	0	1

Component	Component state	
	0	1
1	0.5	0.5
2	0.4	0.6
3	0.1	0.9

Qualitative

Component criticality

Minimal cut/ path sets

Quantitative

Structural importance

Birnbaum's importance

Criticality importance

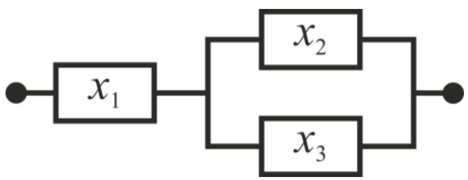
Fussell-Vesely's importance

- W. Kuo and X. Zhu, *Importance Measures in Reliability, Risk, and Optimization: Principles and Applications*. Chichester, UK: Wiley, 2012.

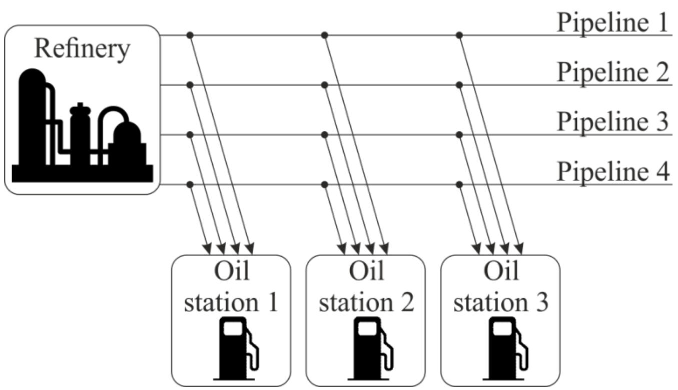
Coherent Systems

- Coherent system:
  - the structure function is non-decreasing in all its arguments

Series-parallel systems



***k*-out-of-*n* systems**

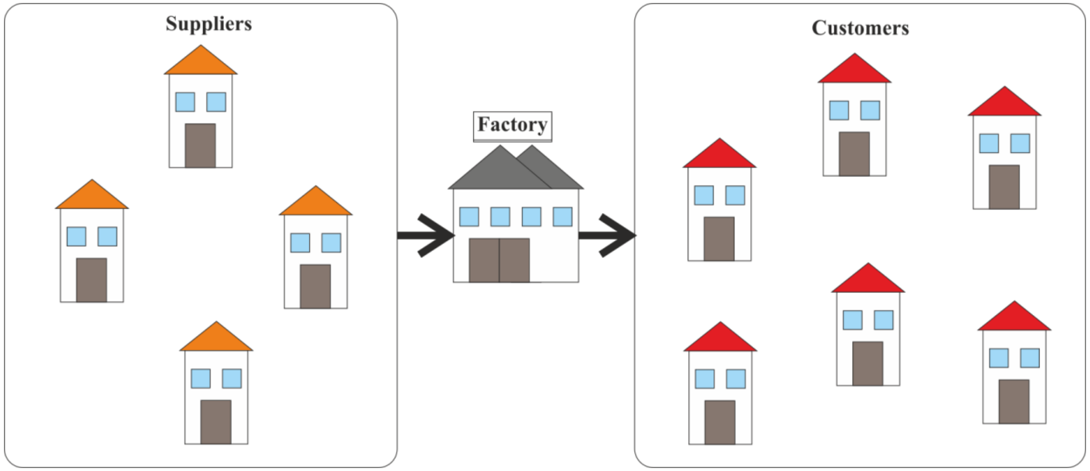


- E. Zaitseva, M. Kvassay, V. Levashenko, and J. Kostolny, “Importance analysis of k-out-of-n multi-state systems based on direct partial logic derivatives,” in *ICTERI 2016*, 2016, pp. 441–457.

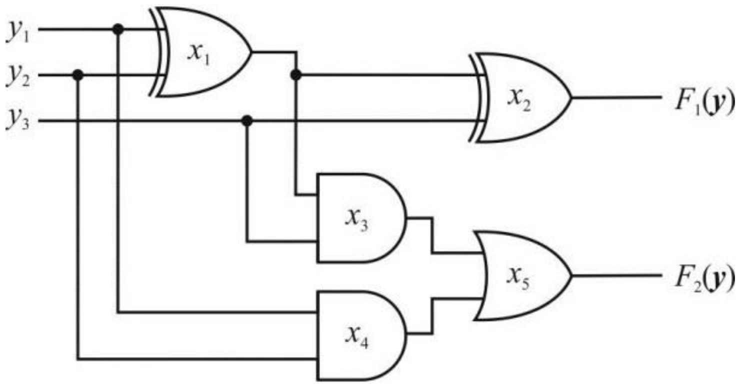
Noncoherent Systems

- Noncoherent system:
  - ~~the structure function is non-decreasing in all its arguments~~

k-to-l-out-of-n systems



Logic circuits



- M. Kvassay, E. Zaitseva, V. Levashenko, and J. Kostolny, “Reliability analysis of multiple-outputs logic circuits based on structure function approach,” *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 36, no. 3, pp. 1–1, Mar. 2016..

Identification of Critical States – Logical Differential Calculus

- Classic partial derivative (real field):

$$\frac{\partial f}{\partial x_i}(x_i) = \lim_{\widetilde{x_i} \to x_i} \frac{f(\widetilde{x_i}, \boldsymbol{x}) - f(x_i, \boldsymbol{x})}{\widetilde{x_i} - x_i}$$

- Boolean partial derivative (GF(2) field):

$$\frac{\partial f}{\partial x_i}(x_i) = f(\overline{x_i}, \boldsymbol{x}) \oplus f(x_i, \boldsymbol{x}) = f(1_i, \boldsymbol{x}) \oplus f(0_i, \boldsymbol{x})$$

- **Let us prove it.**

Logical Differential Calculus

- Some formulae:

$$\frac{\partial \bar{f}}{\partial x_i} = \frac{\partial f}{\partial x_i}$$

$$\frac{\partial c}{\partial x_i} = 0$$

$$\frac{\partial (f \oplus g)}{\partial x_i} = \frac{\partial f}{\partial x_i} \oplus \frac{\partial g}{\partial x_i}$$

$$\frac{\partial (f \wedge g)}{\partial x_i} = g\frac{\partial f}{\partial x_i} \oplus f\frac{\partial g}{\partial x_i} \oplus \frac{\partial f}{\partial x_i}\frac{\partial g}{\partial x_i}$$

$$\frac{\partial (f \vee g)}{\partial x_i} = \bar{g}\frac{\partial f}{\partial x_i} \oplus \bar{f}\frac{\partial g}{\partial x_i} \oplus \frac{\partial f}{\partial x_i}\frac{\partial g}{\partial x_i}$$

- S. N. Yanushkevich, D. M. Miller, V. P. Shmerko, and R. S. Stankovic, *Decision Diagram Techniques for Micro- and Nanoelectronic Design Handbook*, vol. 2. Boca Raton, FL: CRC Press, 2005.

Boolean Derivatives

- Partial logic derivative:

$$\frac{\partial f(\boldsymbol{x})}{\partial x_i} = f(x_i) \oplus f(\overline{x_i}) = f(x_i)\overline{f(\overline{x_i})} \vee \overline{f(x_i)}f(\overline{x_i})$$

- Direct partial logic derivative:

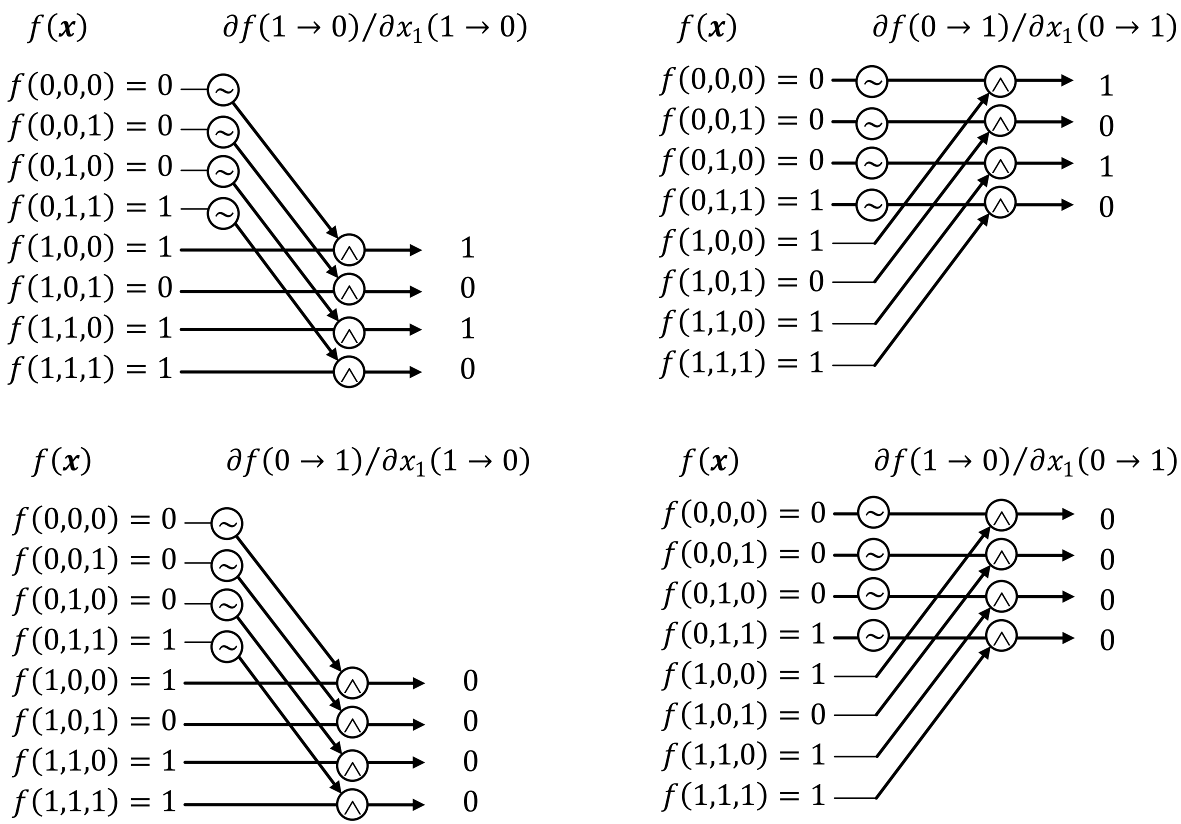
$$\frac{\partial f(1 \to 0)}{\partial x_i(1 \to 0)} = \frac{\partial f(0 \to 1)}{\partial x_i(0 \to 1)} = f(x_i)\overline{f(\overline{x_i})}$$

- Inverse partial logic derivative:

$$\frac{\partial f(1 \to 0)}{\partial x_i(0 \to 1)} = \frac{\partial f(0 \to 1)}{\partial x_i(1 \to 0)} = \overline{f(x_i)}f(\overline{x_i})$$

- S. N. Yanushkevich, D. M. Miller, V. P. Shmerko, and R. S. Stankovic, *Decision Diagram Techniques for Micro- and Nanoelectronic Design Handbook*, vol. 2. Boca Raton, FL: CRC Press, 2005.

Direct Partial Boolean Derivatives

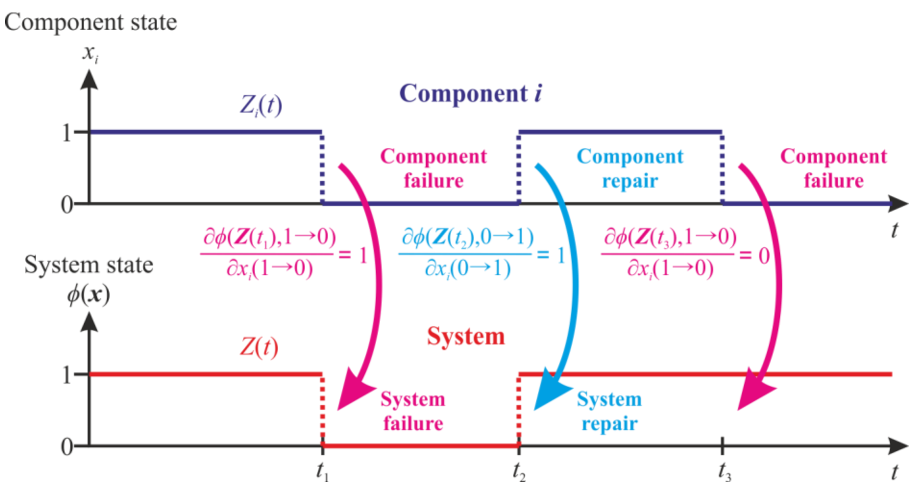


Logical Differential Calculus in Reliability Analysis

$$\frac{\partial \phi(j \to \bar{j})}{\partial x_i(s \to \bar{s})} = \begin{cases} 1 & \text{if } \phi(s_i, \boldsymbol{x}) = j \text{ and } \phi(\bar{s}_i, \boldsymbol{x}) = \bar{j} \\ 0 & \text{otherwise} \end{cases}$$

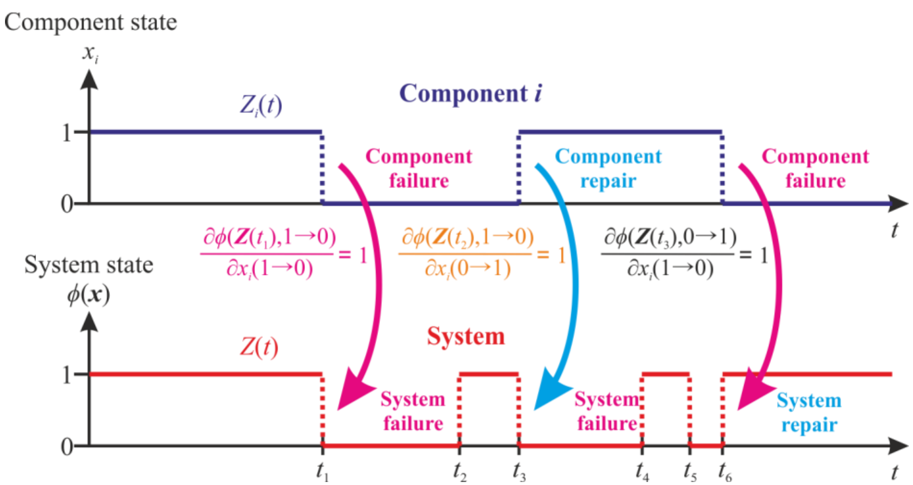
Coherent systems

$$\frac{\partial \phi(1 \to 0)}{\partial x_i(1 \to 0)}, \frac{\partial \phi(0 \to 1)}{\partial x_i(0 \to 1)}$$



Noncoherent systems

$$\frac{\partial \phi(1 \to 0)}{\partial x_i(1 \to 0)}, \frac{\partial \phi(0 \to 1)}{\partial x_i(0 \to 1)}, \frac{\partial \phi(1 \to 0)}{\partial x_i(0 \to 1)}, \frac{\partial \phi(0 \to 1)}{\partial x_i(1 \to 0)}$$



Logical Differential Calculus in Reliability Analysis

Coherent systems

$$\phi(\boldsymbol{x}) = x_1(x_2 \vee x_3)$$

$$\frac{\partial\phi(1\to 0)}{\partial x_3(1\to 0)} = \frac{\partial\phi(0\to 1)}{\partial x_3(0\to 1)} = x_1\overline{x_2}$$

$$\frac{\partial\phi(1\to 0)}{\partial x_3(0\to 1)} = \frac{\partial\phi(0\to 1)}{\partial x_3(1\to 0)} = 0$$

Component	Critical state vectors	Critical path vectors	Critical cut vectors
3	(1,0,.)	(1,0,1)	(1,0,0)

Noncoherent systems

$$\phi(\boldsymbol{x}) = x_1\overline{x_3} \vee x_2 x_3$$

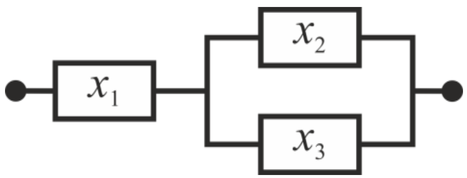
$$\frac{\partial\phi(1\to 0)}{\partial x_3(1\to 0)} = \frac{\partial\phi(0\to 1)}{\partial x_3(0\to 1)} = \overline{x_1}x_2$$

$$\frac{\partial\phi(1\to 0)}{\partial x_3(0\to 1)} = \frac{\partial\phi(0\to 1)}{\partial x_3(1\to 0)} = x_1\overline{x_2}$$

Component	Critical state vectors	State vectors at which failure of component is critical for system failure	State vectors at which repair of component is critical for system failure
3	(0,1,.) (1,0,.)	(0,1,1)	(1,0,0)

Critical State Vectors

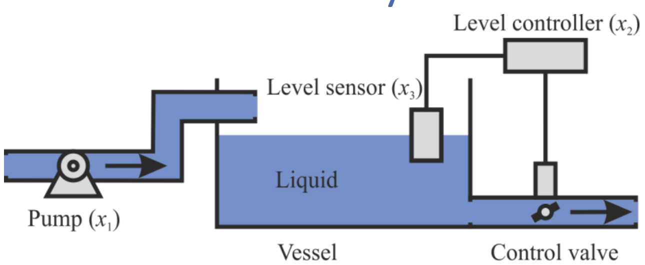
Coherent systems



$$\phi(\boldsymbol{x}) = x_1(x_2 \vee x_3)$$

Component	Critical state vectors	Critical path vectors	Critical cut vectors
1	(.,0,1) (.,1,0) (.,1,1)	(1,0,1) (1,1,0) (1,1,1)	(0,0,1) (0,1,0) (0,1,1)
2	(1,.,0)	(1,1,0)	(1,0,0)
3	(1,0,.)	(1,0,1)	(1,0,0)

Noncoherent systems



$$\phi(\boldsymbol{x}) = x_1\overline{x_3} \vee x_2x_3$$

Component	Critical state vectors	State vectors at which failure of component is critical for system failure	State vectors at which repair of component is critical for system failure
1	(.,0,0) (.,1,0)	(1,0,0) (1,1,0)	
2	(0,.,1) (1,.,1)	(0,1,1) (1,1,1)	
3	(0,1,.) (1,0,.)	(0,1,1)	(1,0,0)

- M. Kvassay, E. Zaitseva, and V. Levashenko, “Reliability analysis of noncoherent systems based on logical differential calculus,” in *Risk, reliability and safety : innovating theory and practice - Proceedings of the European Safety and Reliability Conference, ESREL 2016*, 2017, pp. 1367–1374.

Importance Measures

Coherent systems

$$\phi(\boldsymbol{x}) = x_1(x_2 \vee x_3)$$

$$\mathrm{SI}_3^{\downarrow} = \mathrm{SI}_{3\downarrow}^{\downarrow} = \mathrm{TD}\left(\frac{\partial\phi(1\to 0)}{\partial x_3(1\to 0)}\right) = \mathrm{TD}(x_1\overline{x_2})$$
$$= 0.25$$

$$\mathrm{BI}_3^{\downarrow} = \mathrm{BI}_{3\downarrow}^{\downarrow} = \Pr\left\{\frac{\partial\phi(1\to 0)}{\partial x_3(1\to 0)} = 1\right\} = p_1 q_2$$

Component	Structural importance	Birnbaum's importance
1	0.75	$p_2q_3 + p_3q_2 + p_2p_3$
2	0.25	p_1q_3
3	0.25	p_1q_2

Noncoherent systems

$$\phi(\boldsymbol{x}) = x_1\overline{x_3} \vee x_2x_3$$

$$\mathrm{SI}_3^{\downarrow} = \mathrm{SI}_{3\downarrow}^{\downarrow} + \mathrm{SI}_{3\uparrow}^{\downarrow} = \mathrm{TD}\left(\frac{\partial\phi(1\to 0)}{\partial x_3(1\to 0)}\right) + \mathrm{TD}\left(\frac{\partial\phi(1\to 0)}{\partial x_3(0\to 1)}\right)$$
$$= \mathrm{TD}(\overline{x_1}x_2) + \mathrm{TD}(x_1\overline{x_2}) = 0.25 + 0.25 = 0.5$$

$$\mathrm{BI}_3^{\downarrow} = \mathrm{BI}_{3\downarrow}^{\downarrow} + \mathrm{BI}_{3\uparrow}^{\downarrow} = \Pr\left\{\frac{\partial\phi(1\to 0)}{\partial x_3(1\to 0)} = 1\right\} + \Pr\left\{\frac{\partial\phi(1\to 0)}{\partial x_3(0\to 1)} = 1\right\}$$
$$= q_1p_2 + p_1q_2$$

Component	Structural importance	Birnbaum's importance
1	0.5	q_3
2	0.5	p_3
3	0.5	$p_1q_2+q_1p_2$

InnoSoc

Multiple-Valued Logic Functions and Multi-State Systems

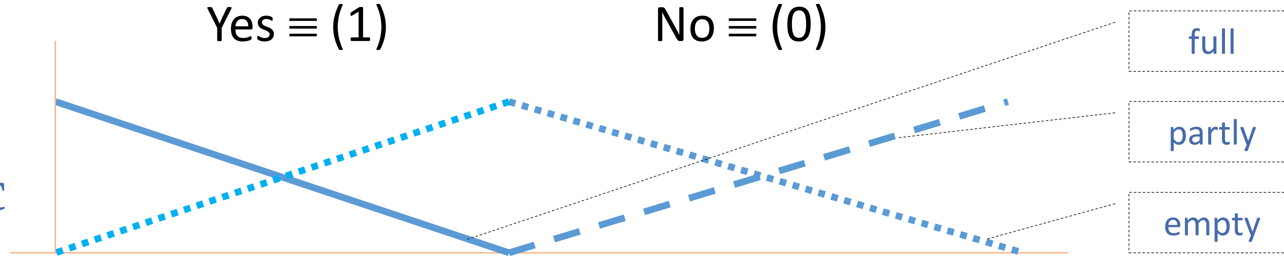
Some Motivations Nonstandard Logics

Is it filled?

Binary Logic

Yes ≡ (1) No ≡ (0)

Fuzzy Logic

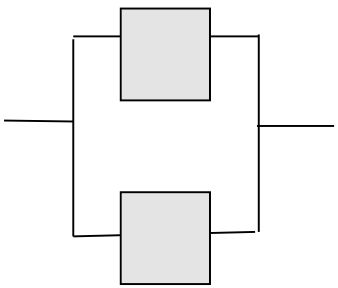


MVL

Yes ≡ (2) It may be ≡ (1) No ≡ (0)

Multi-State System

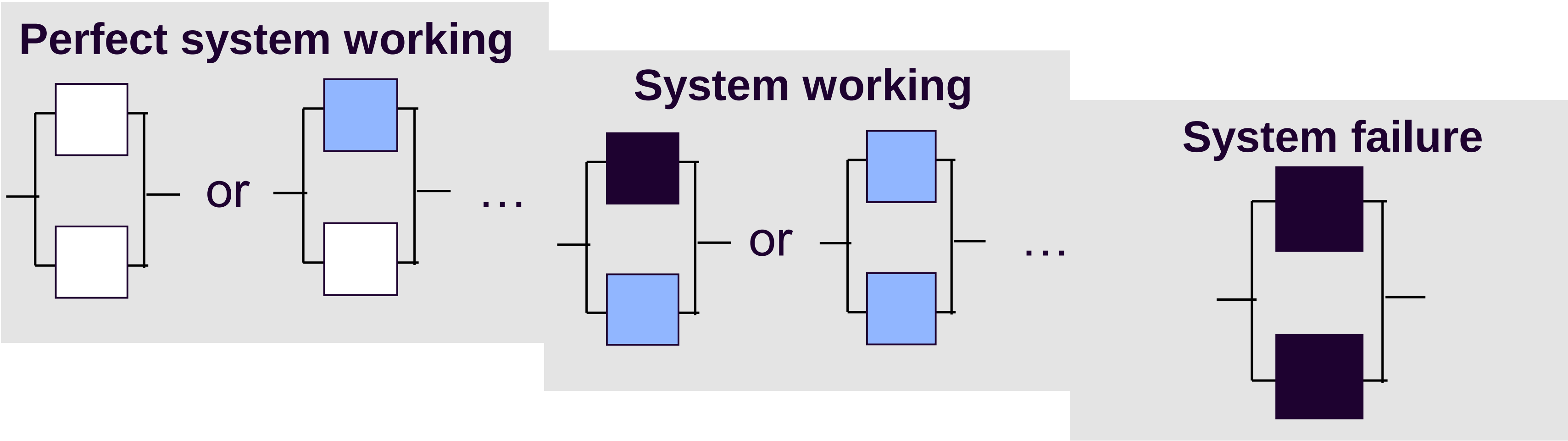
Example of Multi-State System of **two components**



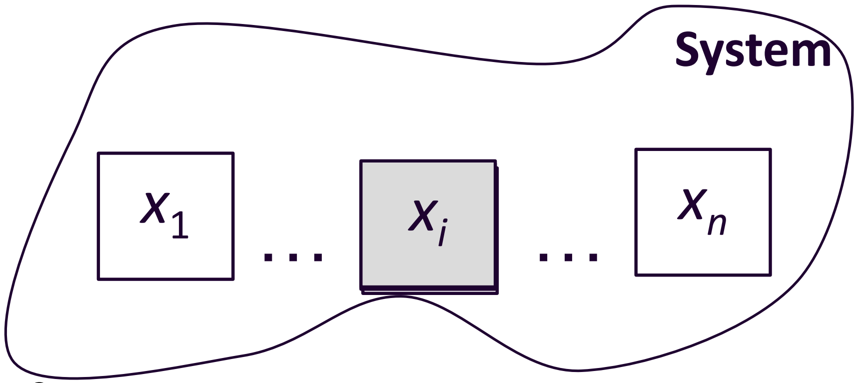
and with **three states** of reliability for the system and its every components

Components states

- perfect working
- working
- breakdown



Multi-State System



- n – number of system components
- m – number of system states
- for i = 1,2,…, n:
  - m_i – number of states of component i
  - x_i – state of component i
    - 0 – component is failed
    - m_i - 1 – component is perfectly functioning
  - $p_{i,s}$ – probability that the i-th component is in state s

- A. Lisnianski, I. Frenkel, and Y. Ding, *Multi-state System Reliability Analysis and Optimization for Engineers and Industrial Managers.* London, UK: Springer-Verlag London Ltd., 2010.

Structure Function

- Structure function defines system topology:

$$\phi(x_1, x_2, \ldots, x_n) = \phi(\boldsymbol{x}): \{0, \ldots, m_1\text{-}1\}\times\ldots\times\{0, \ldots, m_n\text{-}1\} \rightarrow \{0, \ldots, m\text{-}1\}$$

$$m_1 = m_2 = \ldots = m_n = m \Rightarrow \text{homogeneous system}$$

Structure function

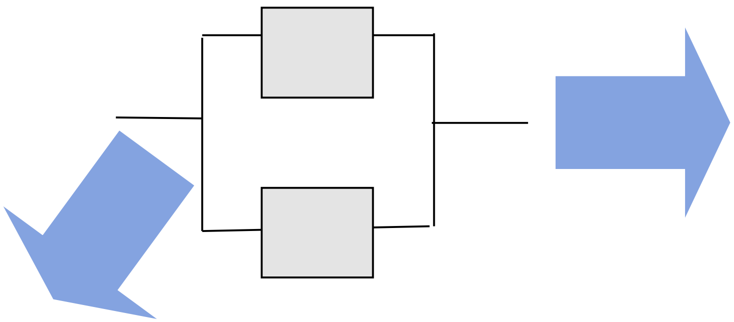
- state of the system at a fixed time
- state of component *i* at the fixed time

Logic function

- function value
- value of the *i*-th variable of the function

Tools of multiple-valued logic can be used in reliability analysis of multi-state systems

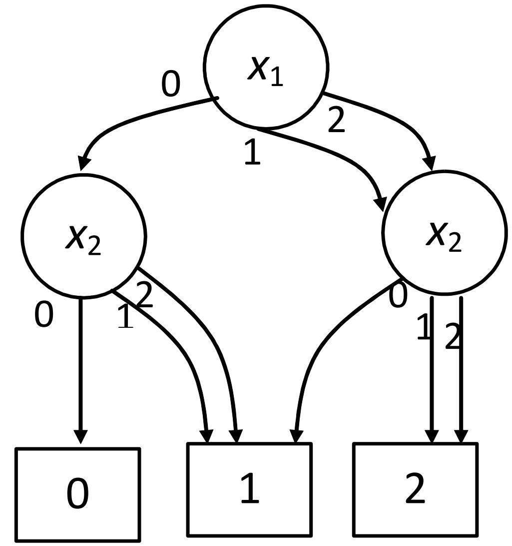
Different Interpretation of Multi-State Systems



Truth table of structure function

$x_1\ x_2$	$\phi(\boldsymbol{x})$
0 0	0
0 1	1
0 2	1
1 0	1
1 1	2
1 2	2
2 0	1
2 1	2
2 2	2

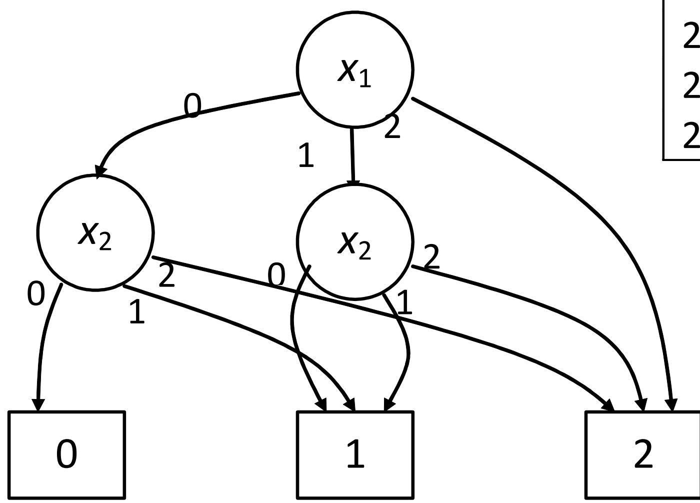
The structure function MDD



Truth table of structure function

$x_1\ x_2$	$\phi(\boldsymbol{x})$
0 0	0
0 1	1
0 2	2
1 0	1
1 1	1
1 2	2
2 0	2
2 1	2
2 2	2

The structure function MDD



Basic Characteristics

- Component states probabilities:

$$p_{i,s} = \Pr\{x_i = s\}, \qquad i \in \{1,2,\dots,n\}, s \in \{0,1,\dots,m_i - 1\}$$

- System state probability:

$$\Pr\{\phi(\boldsymbol{x}) = j\}, \qquad j \in \{0,1,\dots,m-1\}$$

- System availability/unavailability:

$$\begin{aligned} A^{\geq j} &= \Pr\{\phi(\boldsymbol{x}) \geq j\} \\ U^{\geq j} &= \Pr\{\phi(\boldsymbol{x}) < j\}, \end{aligned} \qquad j \in \{1,2,\dots,m-1\}$$

- Performance utility function:

$$O = \sum_{j=0}^{m-1} o_j \Pr\{\phi(\boldsymbol{x}) = j\},$$

o_j – utility attached to state j

Importance Analysis

- Influence of:
  - given component state on given system state / availability level
  - given component on given system state / availability level
  - given component state on the whole system
  - given component on the whole system

Qualitative

Component criticality

Minimal cut/ path sets

Quantitative

Structural importance

Birnbaum's importance

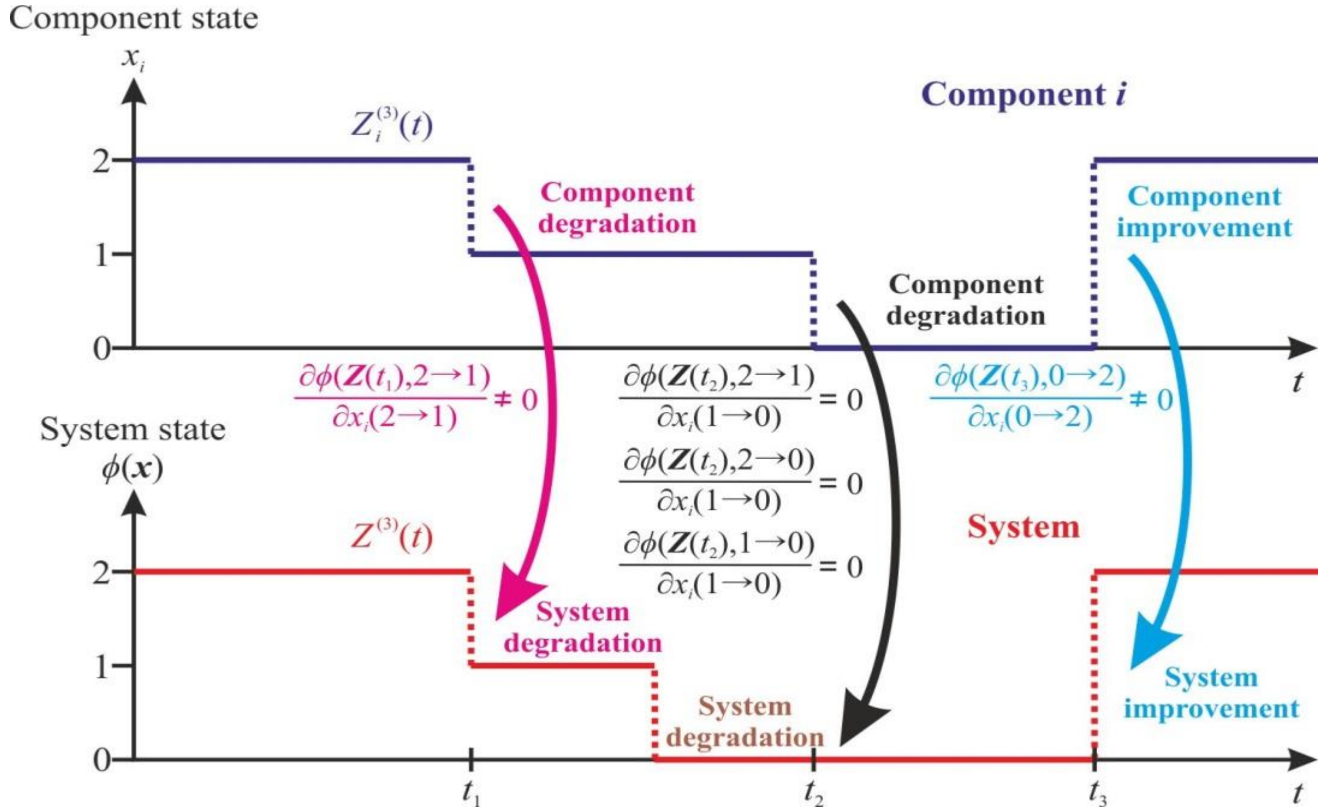
Criticality importance

Fussell-Vesely's importance

- M. Kvassay, E. Zaitseva, and V. Levashenko, "Importance analysis of multi-state systems based on tools of logical differential calculus," *Reliability Engineering & System Safety*, vol. 165, no. December 2016, pp. 302–316, Sep. 2017.

Logical Differential Calculus

$$\frac{\partial\phi(j \to h)}{\partial x_i(s \to r)} = \begin{cases} 1 & \text{if } \phi(s_i, \boldsymbol{x}) = j \text{ and } \phi(r_i, \boldsymbol{x}) = h \\ 0 & \text{otherwise} \end{cases}$$



Direct Partial Logic Derivatives

$$\frac{\partial\phi(j \to h)}{\partial x_i(s \to r)} = \begin{cases} 1 & \text{if } \phi(s_i, \boldsymbol{x}) = j \text{ and } \phi(r_i, \boldsymbol{x}) = h \\ 0 & \text{otherwise} \end{cases}$$

Integrated Direct Partial Logic Derivatives:

Type I:

$$\frac{\partial\phi(j \searrow)}{\partial x_i(s \to r)} = \begin{cases} 1 & \text{if } \phi(s_i, \boldsymbol{x}) = j \text{ and } (r_i, \boldsymbol{x}) < j \\ 0 & \text{otherwise} \end{cases}$$

Type II:

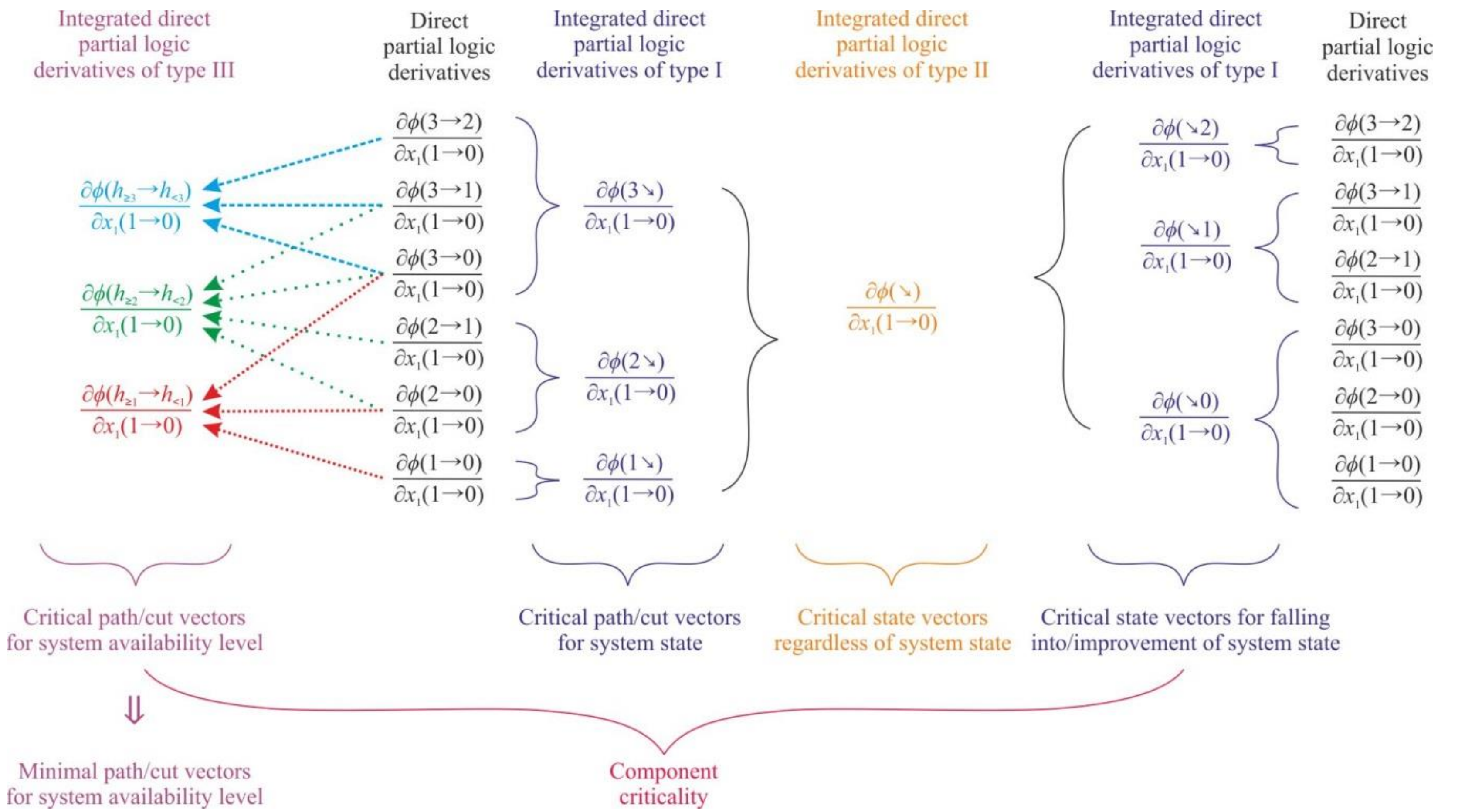
$$\frac{\partial\phi(\searrow)}{\partial x_i(s \to r)} = \begin{cases} 1 & \text{if } \phi(s_i, \boldsymbol{x}) > (r_i, \boldsymbol{x}) \\ 0 & \text{otherwise} \end{cases}$$

Type III:

$$\frac{\partial\phi\left(h_{\geq j} \to h_{<j}\right)}{\partial x_i(s \to r)} = \begin{cases} 1 & \text{if } \phi(s_i, \boldsymbol{x}) \geq j \text{ and } (r_i, \boldsymbol{x}) < j \\ 0 & \text{otherwise} \end{cases}$$

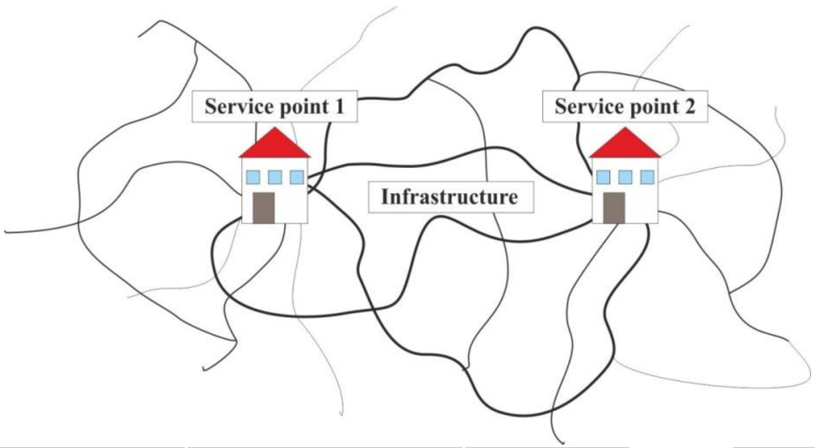
- M. Kvassay, E. Zaitseva, and V. Levashenko, “Importance analysis of multi-state systems based on tools of logical differential calculus,” *Reliability Engineering & System Safety*, vol. 165, no. December 2016, pp. 302–316, Sep. 2017.

Relations between DPLDs



- M. Kvassay, E. Zaitseva, and V. Levashenko, "Importance analysis of multi-state systems based on tools of logical differential calculus," *Reliability Engineering & System Safety*, vol. 165, no. December 2016, pp. 302–316, Sep. 2017.

Example of Structural Importance



$\phi(x_1, x_2, x_3)$		x_3			
x_1	x_2	0	1	2	3
0	0	0	0	0	0
0	1	0	1	1	2
1	0	0	1	1	2
1	1	0	2	3	3

Component 1		Component state		Average
		0	1	
System state	0	–	–	–
	1	–	0.25	0.25
	2	–	0.25	0.25
	3	–	0.25	0.25
Sum		–	0.75	0.75

Component 3		Component state				Average
		0	1	2	3	
System state	0	–	–	–	–	–
	1	–	0.50	0	0	0.1666
	2	–	0.25	0	0.50	0.2500
	3	–	0	0.25	0	0.0833
Sum		–	0.75	0.25	0.50	0.50

- M. Kvassay, E. Zaitseva, J. Kostolny, and V. Levashenko, “Importance analysis of multi-state systems based on integrated direct partial logic derivatives,” in *2015 International Conference on Information and Digital Technologies (IDT)*, 2015, pp. 183–195.

Data Mining

Knowledge Discovery Process

1. Understanding the problem domain
2. Understanding the data
3. Preparation of the data
4. **Data mining**
5. Evaluation of the discovered knowledge
6. Using the discovered knowledge

Knowledge discovery is not a linear process.

- K. J. Cios and G. W. Moore, "Medical data mining and knowledge discovery: Overview of key issues," in *Medical Data Mining and Knowledge Discovery*, K. J. Cios, Ed. New York, NY: Physica Verlag Heidelberg, 2001, pp. 1–20.

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Understanding the Data

- What data is available?
- Which data will be used?
- Which additional information will be needed?

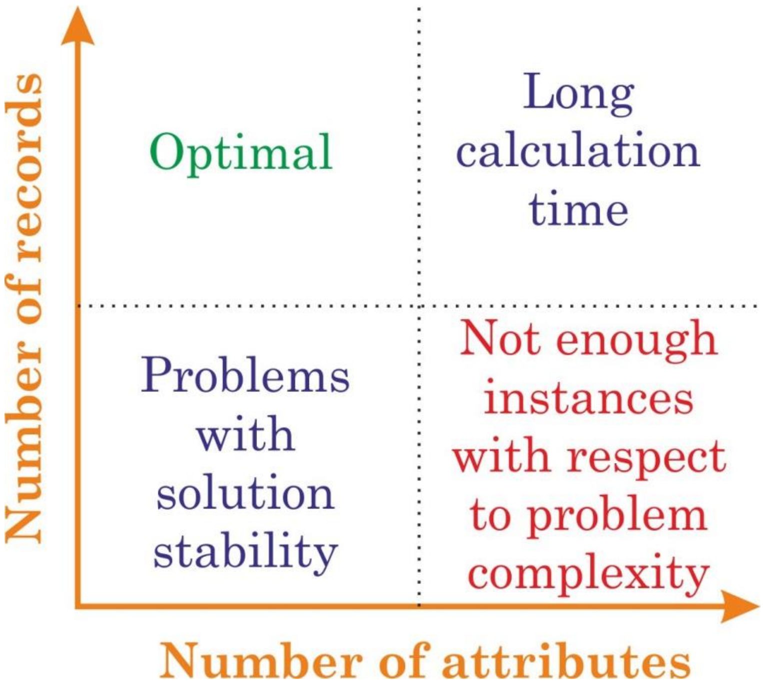
create a target data set

columns (attributes)

rows (records)

No.	Tumor	History	Heredity	Age	Cancer
1	confirmed	high	yes	younger	high
2	confirmed	high	yes	elder	high
3	no	high	yes	younger	low
4	non confirmed	medium	yes	younger	low
5	non confirmed	low	no	younger	low
6	non confirmed	low	no	elder	high
7	no	low	no	elder	low
8	confirmed	medium	yes	younger	high
9	confirmed	low	no	younger	low
10	non confirmed	medium	no	younger	low
11	confirmed	medium	no	elder	low
12	no	medium	yes	elder	low
13	no	high	no	younger	low
14	non confirmed	medium	yes	elder	high

How Much Data?



Linguistic Data

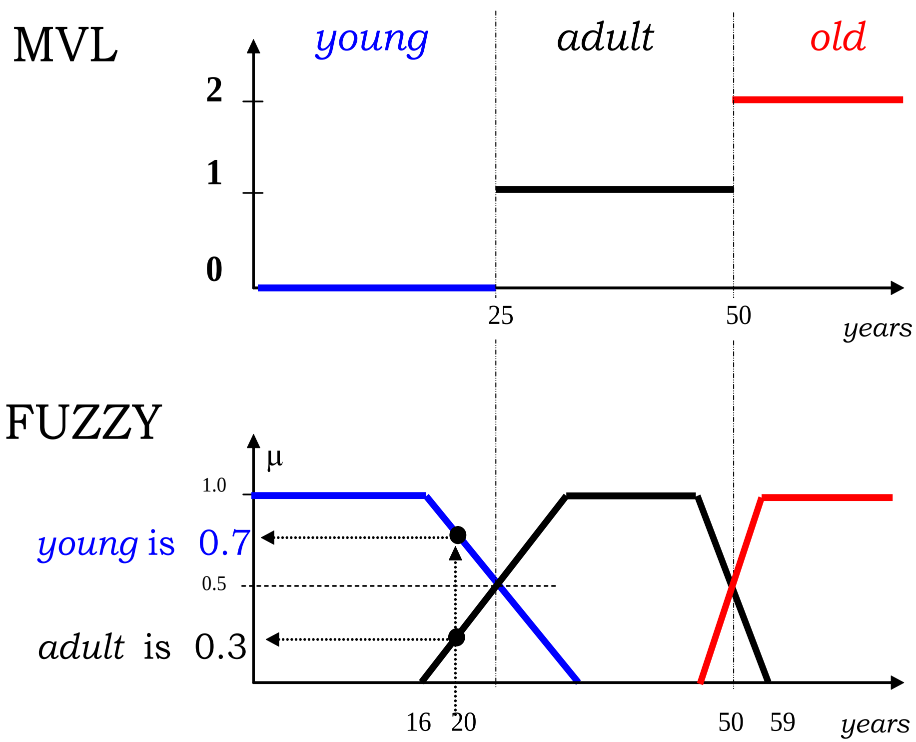
- data mining – sophisticated process with inaccurate data
- linguistic data transforms inaccuracy into vague
- linguistic data is simpler for understanding (models are smaller and simpler)

Experience	
Nurse	**Doctor**
1	1
1	3
2	1
2	2
4	1
5	4

Age (numerical)
8
20
25
40
50
75

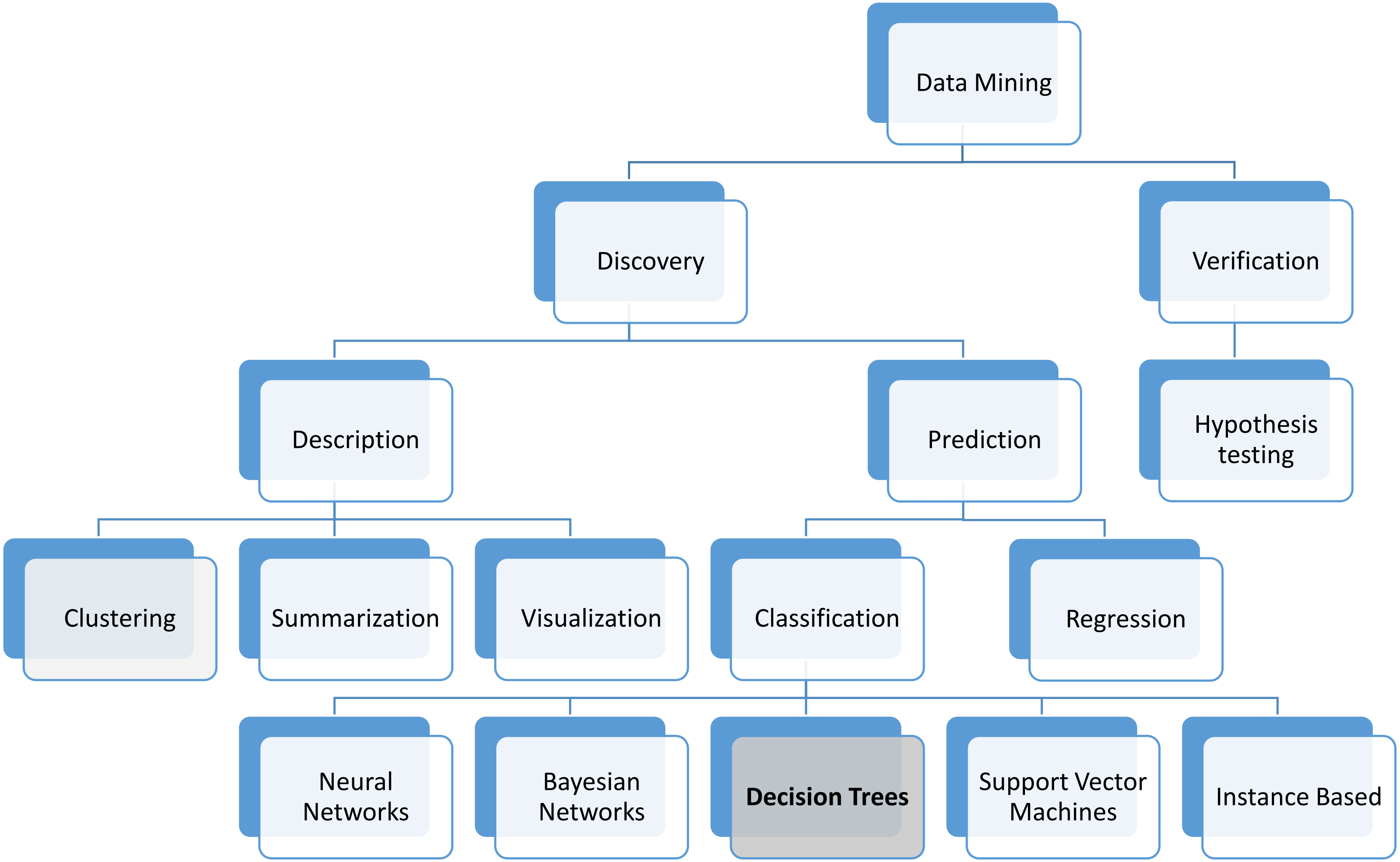
Age (linguistic)
young
young
adult
adult
adult
old

Multiple-Valued and Fuzzy Data



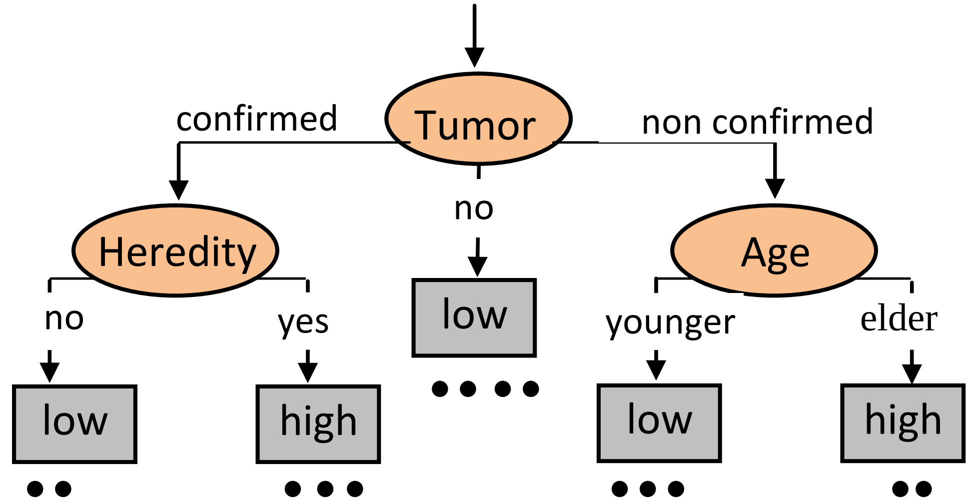
Numeric A_*	Linguistic attribute A_* (*Age*)		
	$A_{*,1}$(*young*)	$A_{*,2}$ (*adult*)	$A_{*,3}$ (*old*)
x_1 = 8	1.0	0.0	0.0
x_2 = **20**	**0.7**	**0.3**	**0.0**
x_3 = 25	0.5	0.5	0.0
x_4 = 40	0.0	1.0	0.0
x_5 = 50	0.0	0.5	0.5
x_6 = 75	0.0	0.0	1.0

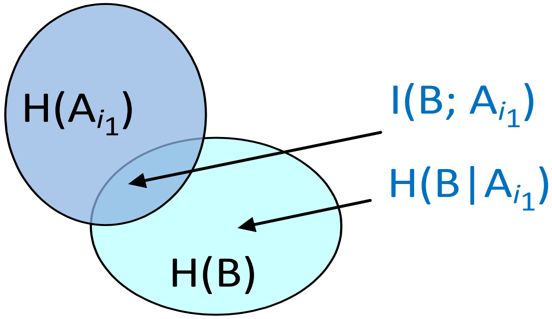
Data Mining


Data Mining
Discovery
Verification
Description
Prediction
Hypothesis testing
Clustering
Summarization
Visualization
Classification
Regression
Neural Networks
Bayesian Networks
Decision Trees
Support Vector Machines
Instance Based

Decision Trees

No.	Tumor	A2	Heredity	Age	Cancer
1	confirmed	85	yes	younger	high
2	confirmed	80	yes	elder	high
3	no	83	yes	younger	low
4	non confirr	70	yes	younger	low
5	non confirr	68	no	younger	low
6	non confirr	65	no	elder	high
7	no	64	no	elder	low
8	confirmed	72	yes	younger	high
9	confirmed	69	no	younger	low
10	non confirr	75	no	younger	low
11	confirmed	75	no	elder	low
12	no	72	yes	elder	low
13	no	81	no	younger	low
14	non confirr	71	yes	elder	high





$$H(B) = I(B; A) + H(B \mid A)$$

$H(B)$ - describes the uncertainty of attribute B

$H(B|A_i)$ - describes the uncertainty of attribute B when the attribute A_i is given

$I(B ; A_i)$ - is used as to measure the dependence of the attribute B on the attribute A_i and vice-versa

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Application of Data Mining in Construction of Structure Function

Laparoscopic Surgery Procedure

- **System**
  - 0 – non-operational (fatal medical error),
  - 1 – partially operational (some imperfection),
  - 2 – fully operational (surgery without any complication).
- **Device** (m_1 = 2):
  - 0 – failure, and
  - 1 –functioning.
- **Work of anesthesiologist** (m_2 = 2):
  - 0 – non-operational (medical error),
  - 1 – fully operational (without any complication).
- **Work of surgeon and the nurse** (m_3 = m_4 = 3), i.e.:
  - 0 – (the fatal error),
  - 1 – (sufficient), and
  - 2 – (perfect or the work without any complication).

- V. Levashenko, E. Zaitseva, M. Kvassay, and T. M. Deserno, "Reliability estimation of healthcare systems using Fuzzy Decision Trees," in *2016 Federated Conference on Computer Science and Information Systems (FedCSIS)*, 2016, pp. 331–340.

Collection of Data in the Repository

No	x_1		x_2		x_3			x_4			$\phi(x)$		
	0	**1**	**0**	**1**	**0**	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
1	0.6	0.4	0.9	0.1	0.1	0.9	0.0	0.2	0.6	0.2	0.9	0.1	0.0
2	0.7	0.3	1.0	0.0	0.0	0.9	0.1	0.1	0.8	0.1	0.8	0.1	0.1
3	0.5	0.5	0.9	0.1	0.8	0.2	0.0	0.8	0.1	0.1	0.9	0.1	0.0
4	1.0	0.0	0.1	0.9	1.0	0.0	0.0	0.1	0.9	0.0	0.8	0.2	0.0
5	0.9	0.1	0.0	1.0	0.1	0.2	0.0	0.1	0.9	0.0	1.0	0.0	0.0
6	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	1.0	0.0
7	1.0	0.0	0.0	1.0	0.0	0.1	0.9	0.0	0.3	0.7	0.1	0.8	0.1
8	0.0	1.0	1.0	0.0	0.0	1.0	0.0	0.0	0.6	0.6	0.1	0.9	0.0
9	0.1	0.9	0.1	0.9	0.1	0.1	0.8	1.0	0.0	0.0	0.1	0.8	0.1
10	0.3	0.7	0.9	0.1	0.0	0.0	1.0	0.0	0.5	0.5	0.0	0.1	0.0
11	0.2	0.8	0.0	1.0	0.9	0.1	0.0	0.0	1.0	0.0	1.0	0.0	0.0
12	0.0	1.0	0.0	1.0	0.1	0.9	0.0	0.8	0.2	0.0	0.0	1.0	0.0
13	0.1	0.9	0.2	0.9	0.1	0.8	0.1	0.0	0.6	0.4	0.0	0.0	1.0
14	0.2	0.8	0.0	1.0	0.0	0.1	0.9	1.0	0.0	0.0	0.1	0.8	0.1
15	0.3	0.7	0.0	1.0	0.0	0.1	0.9	0.1	0.8	0.1	0.0	0.1	0.9

- V. Levashenko, E. Zaitseva, M. Kvassay, and T. M. Deserno, “Reliability estimation of healthcare systems using Fuzzy Decision Trees,” in *2016 Federated Conference on Computer Science and Information Systems (FedCSIS)*, 2016, pp. 331–340.

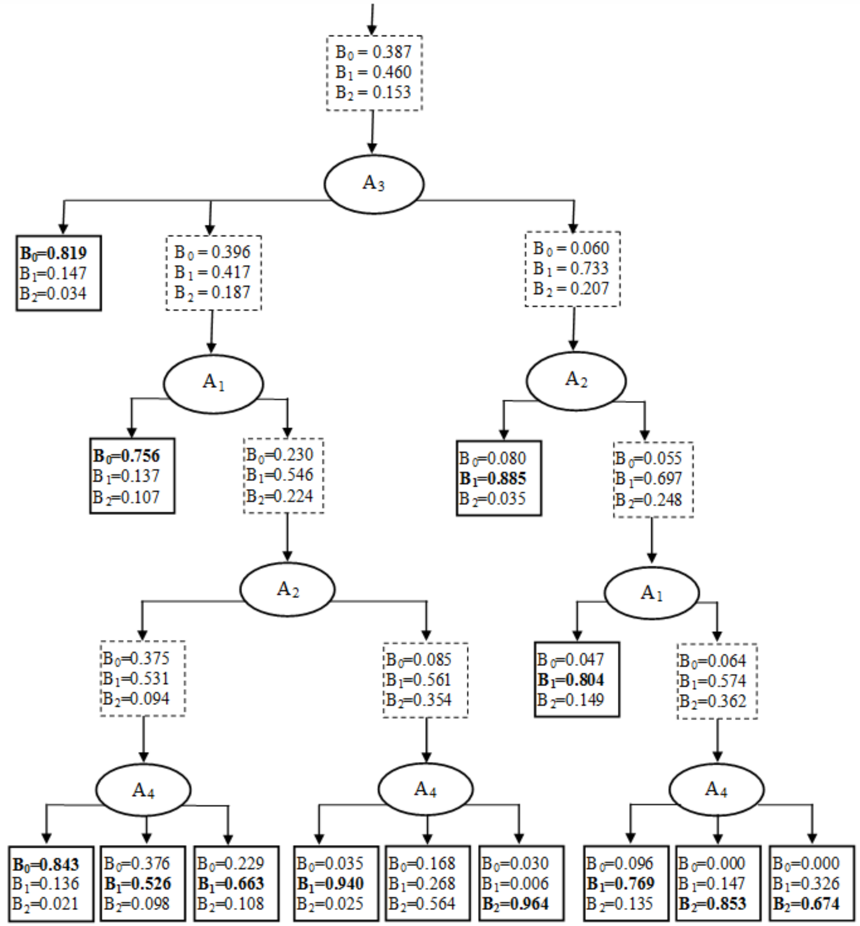
Dataset and Structure Function

FDT	System reliability
Number of input attribute: n	Number of the system components: n
Attribute A_i ($i = 1, \ldots, n$)	System component x_i ($i = 1, \ldots, n$)
Attribute A_i values: $\{A_{i,0}, \ldots, A_{i,j}, \ldots, A_{i,m_i-1}\}$	The i-th system component state: $\{0, \ldots, m_i-1\}$
Output attribute B	System performance level $\phi(\boldsymbol{x})$
Values of output attribute B: $\{B_0, \ldots, B_{M-1}\}$	System performance level values: $\{0, \ldots, M-1\}$
Decision table	Structure function

- V. Levashenko, E. Zaitseva, M. Kvassay, and T. M. Deserno, “Reliability estimation of healthcare systems using Fuzzy Decision Trees,” in *2016 Federated Conference on Computer Science and Information Systems (FedCSIS)*, 2016, pp. 331–340.

Construction of Fuzzy Decision Tree

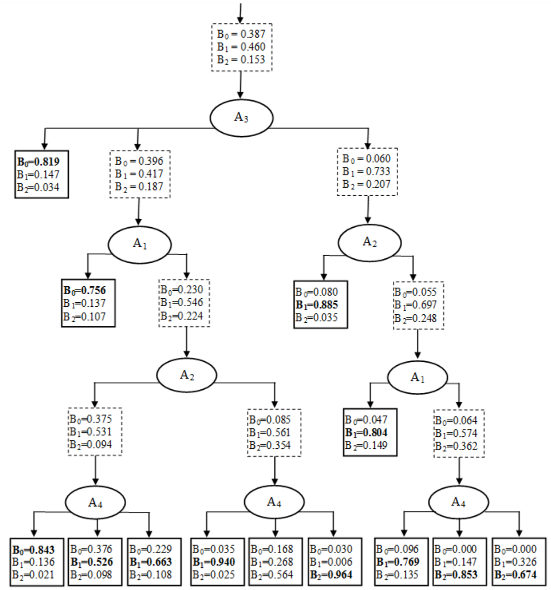
No	x_1		x_2		x_3		
	0	**1**	**0**	**1**	**0**	**1**	**2**
1	0.6	0.4	0.9	0.1	0.1	0.9	0.0
2	0.7	0.3	1.0	0.0	0.0	0.9	0.1
3	0.5	0.5	0.9	0.1	0.8	0.2	0.0
4	1.0	0.0	0.1	0.9	1.0	0.0	0.0
5	0.9	0.1	0.0	1.0	0.1	0.2	0.0
6	1.0	0.0	0.0	1.0	0.0	0.0	1.0
7	1.0	0.0	0.0	1.0	0.0	0.1	0.9
8	0.0	1.0	1.0	0.0	0.0	1.0	0.0
9	0.1	0.9	0.1	0.9	0.1	0.1	0.8
10	0.3	0.7	0.9	0.1	0.0	0.0	1.0
11	0.2	0.8	0.0	1.0	0.9	0.1	0.0
12	0.0	1.0	0.0	1.0	0.1	0.9	0.0
13	0.1	0.9	0.2	0.9	0.1	0.8	0.1
14	0.2	0.8	0.0	1.0	0.0	0.1	0.9
15	0.3	0.7	0.0	1.0	0.0	0.1	0.9



- V. Levashenko, E. Zaitseva, M. Kvassay, and T. M. Deserno, “Reliability estimation of healthcare systems using Fuzzy Decision Trees,” in *2016 Federated Conference on Computer Science and Information Systems (FedCSIS)*, 2016, pp. 331–340.

Structure Function Generation

No	x_1		x_2		x_3			x_4			$\phi(x)$		
	0	**1**	**0**	**1**	**0**	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
1	0.6	0.4	0.9	0.1	0.1	0.9	0.0	0.2	0.6	0.2	0.9	0.1	0.0
2	0.7	0.3	1.0	0.0	0.0	0.9	0.1	0.1	0.8	0.1	0.8	0.1	0.1
3	0.5	0.5	0.9	0.1	0.8	0.2	0.0	0.8	0.1	0.1	0.9	0.1	0.0
4	1.0	0.0	0.1	0.9	1.0	0.0	0.0	0.1	0.9	0.0	0.8	0.2	0.0
5	0.9	0.1	0.0	1.0	0.1	0.2	0.0	0.1	0.9	0.0	1.0	0.0	0.0
6	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	1.0	0.0
7	1.0	0.0	0.0	1.0	0.0	0.1	0.9	0.0	0.3	0.7	0.1	0.8	0.1
8	0.0	1.0	1.0	0.0	0.0	1.0	0.0	0.0	0.6	0.6	0.1	0.9	0.0
9	0.1	0.9	0.1	0.9	0.1	0.1	0.8	1.0	0.0	0.0	0.1	0.8	0.1
10	0.3	0.7	0.9	0.1	0.0	0.0	1.0	0.0	0.5	0.5	0.0	0.1	0.0
11	0.2	0.8	0.0	1.0	0.9	0.1	0.0	0.0	1.0	0.0	1.0	0.0	0.0
12	0.0	1.0	0.0	1.0	0.1	0.9	0.0	0.8	0.2	0.0	0.0	1.0	0.0
13	0.1	0.9	0.2	0.9	0.1	0.8	0.1	0.0	0.6	0.4	0.0	0.0	1.0
14	0.2	0.8	0.0	1.0	0.0	0.1	0.9	1.0	0.0	0.0	0.1	0.8	0.1
15	0.3	0.7	0.0	1.0	0.0	0.1	0.9	0.1	0.8	0.1	0.0	0.1	0.9



Component states	x_1	0	0	0	0	0	0	0
	x_2	0	0	0	0	0	0	0
	x_3	0	0	0	1	1	1	[illegible]
	x_4	0	1	2	0	1	2	
System performance level	$\phi(x)$	**0**	**0**	**1**	**0**	**1**	**1**	

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Reliability Analysis

No	x_1		x_2		x_3			x_4			$\phi(x)$		
	0	**1**	**0**	**1**	**0**	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
1	0.6	0.4	0.9	0.1	0.1	0.9	0.0	0.2	0.6	0.2	0.9	0.1	0.0
2	0.7	0.3	1.0	0.0	0.0	0.9	0.1	0.1	0.8	0.1	0.8	0.1	0.1
3	0.5	0.5	0.9	0.1	0.8	0.2	0.0	0.8	0.1	0.1	0.9	0.1	0.0
4	1.0	0.0	0.1	0.9	1.0	0.0	0.0	0.1	0.9	0.0	0.8	0.2	0.0
5	0.9	0.1	0.0	1.0	0.1	0.2	0.0	0.1	0.9	0.0	1.0	0.0	0.0
6	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	1.0	0.0
7	1.0	0.0	0.0	1.0	0.0	0.1	0.9	0.0	0.3	0.7	0.1	0.8	0.1
8	0.0	1.0	1.0	0.0	0.0	1.0	0.0	0.0	0.6	0.6	0.1	0.9	0.0
9	0.1	0.9	0.1	0.9	0.1	0.1	0.8	1.0	0.0	0.0	0.1	0.8	0.1
10	0.3	0.7	0.9	0.1	0.0	0.0	1.0	0.0	0.5	0.5	0.0	0.1	0.0
11	0.2	0.8	0.0	1.0	0.9	0.1	0.0	0.0	1.0	0.0	1.0	0.0	0.0
12	0.0	1.0	0.0	1.0	0.1	0.9	0.0	0.8	0.2	0.0	0.0	1.0	0.0
13	0.1	0.9	0.2	0.9	0.1	0.8	0.1	0.0	0.6	0.4	0.0	0.0	1.0
14	0.2	0.8	0.0	1.0	0.0	0.1	0.9	1.0	0.0	0.0	0.1	0.8	0.1
15	0.3	0.7	0.0										

System component description	Component's states probabilities		
	$p_{i,2}$	$p_{i,1}$	$p_{i,0}$
The laparoscopic robotic surgery machine functioning, x_1	—	0.98	0.02
The anesthesiologist's work, x_2	—	0.94	0.06
The surgeon's work, x_3	0.64	0.27	0.09
The nurse's work, x_4	0.47	0.35	0.18

a) fatal medical error with probability 0.098,

b) sufficient result (some complications) with probabilities 0.214 and

c) perfect result 0.688 (without any complications).

- V. Levashenko, E. Zaitseva, M. Kvassay, and T. M. Deserno, “Reliability estimation of healthcare systems using Fuzzy Decision Trees,” in *2016 Federated Conference on Computer Science and Information Systems (FedCSIS)*, 2016, pp. 331–340.

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